Secondary

MATHEMATICS

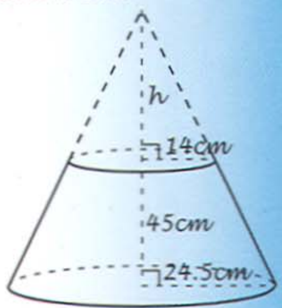
Form Two Teachers' Guide **Third Edition**

Question

A commemorative stone is sculptured in a shape of a frustum with the diameters of the top and bottom faces being 28 cm and 49 cm respectively. If the vertical distance between the faces is 45 cm, find the volume of the stone.

Solution

The two cones in the figure are similar.
Let the height of the smaller cone be h cm.



$$\frac{14}{24.5} = \frac{h}{h+45}$$

$$24.5h = 14(h+45)$$

$$24.5h - 14h = 630$$

$$h = \frac{630}{10.5}$$

$$= 60\text{ cm}$$

Volume of the stone = volume of larger cone − volume of smaller cone

$$= \frac{1}{3} \times \frac{22}{7} \times 24.5^2 \times 105 - \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 60$$

KENYA LITERATURE BUREAU

Approved by the Ministry of Education

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Chapter One

CUBES AND CUBE ROOTS

This is a new topic to the learner. However, the learner has met situations where the cube of a number is required, as in calculations involving volume.

Objectives

By the end of the topic, the learner should be able to:

(i) find the cube of a number by multiplication.
(ii) find the cube of a number using mathematical tables.
(iii) find cube root of a number using the factor method.
(iv) evaluate expressions involving cubes and cube roots.
(v) apply the knowledge of cubes and cube roots in real-life situations.

Time: Fourteen lessons.

Teaching/Learning Activities

Cubes of Numbers by Multiplication

- The teacher should involve the learner in review of squares of numbers. The learner should then be involved in multiplying a number thrice repeatedly, leading to the understanding of the meaning of a cube of a number as in the students' book.
- The teacher should discuss examples 1 and 2.
- The learner should be given exercise 1.1.

Cube of a Number using Mathematical Tables

- The learner should be involved in revision of finding squares using mathematical tables.
- The teacher should discuss example 3 of the students' book.
- The teacher should give further examples on finding cubes of numbers up to 4 significant figures.
- The learner should be given exercise 1.2 of the students' book.

Cube Root of a Number by Factor Method

- The teacher should involve the learner in revision of expressing numbers as products of prime factors.
- The learner should be guided through examples 4, 5 and 6.
- The learner should be given exercise 1.3.

Evaluation of Expressions Involving Cubes and Cube Roots

- The learner should be given problems involving evaluating cubes and cube roots of expressions, as in example 5.
- The learner should be given question 2 of exercise 1.3.
- The teacher should give a speed exercise in addition to exercise 1.3.

Application of Cubes and Cube Roots in Real Life Situations

The learner should be guided through real life situation problems, as in questions 4 to 7 of exercise 1.3.

Additional Hints

(i) Algebraic expressions usually give learners difficulties. Ample practice should therefore be provided on this, e.g.;

$$(a^3b^2)^3 = a^3b^2 \times a^3b^2 \times a^3b^2$$
$$= a^9b^6$$

$$\text{or } (a^3b^2)^3 = (a^3)^3 \times (b^2)^3$$
$$= a^9 \times b^6$$
$$= a^9 b^6$$

$$\sqrt[3]{(x^6y^9)} = \sqrt[3]{x^6} \times \sqrt[3]{y^9}$$
$$= x^2 \times y^3$$
$$= x^2y^3$$

(ii) The teacher should give examples involving cubes and cube roots of negative directed numbers. For example;

$$(-5)^3 = -5 \times -5 \times -5$$
$$= -125$$

$$\sqrt[3]{-(64)} = {}^{-}\sqrt[3]{64}$$
$$= -4$$

(iii) Problems involving transformations (change of shape) should be emphasised. The fact that the volume of the material remains the same, see question 7 exercise 1.3, should be made clear.

(iv) The teacher may give a hint on how to find cube roots of particular numbers using mathematical tables, as in table 1.1. For example;

$$\sqrt[3]{70.44} = 4.130.$$

Evaluation

- Give a written test covering cubes and cube roots. Go through the test with the learners to ensure all concepts are understood clearly.
- To probe learners' understanding further, write random figures on the chalkboard and ask them to find cubes and cube roots. Give the learners a chance to demonstrate on the chalkboard.

Further Questions

1. (a) $\sqrt[3]{3^{27}}$ (b) $\sqrt[3]{(27 \times 64)}$ (c) $\sqrt[3]{(3^{12} \times 5^{6} \times 7^{3})}$

2. Find the cube roots of:
   (a) 2 744 (b) 21 952 (c) 91 125

Answers

Exercise 1.1

1. (a) 216 (b) 4.096 (c) 32.77 (4 s.f.)
2. 10.22 (4 s.f.)
3. (a) 0.064 (b) 0.125 (c) 1.331 (d) 0.000027
4. 27 cm^3
5. (a) a^3b^3 (b) x^3y^3 (c) $8b^3$

Exercise 1.2

1. (a) 571.8 (b) 1.030 (c) 15.70 (4 s.f.)
   (d) 0.6585 (4 s.f.) (e) 3 688
2. (a) 66.92 (b) 251.5 (c) 0.0007586
   (d) 55 611 740 (e) 9.986 (f) 154.7
3. 9 261 cm^3
4. 9.842 *l*

Exercise 1.3

1. (a) 4 (b) 5 (c) 15
   (d) $\frac{3}{4}$ (e) 0.8 (f) $\frac{7}{5}$ or $1\frac{2}{5}$
2. (a) ab (b) x^3y^3 (c) w^2y (d) $3xy^3$
3. (a) $\frac{2}{7}$ (b) 7.56 (c) 15
4. 6.3 cm
5. 12 cm
6. (a) 343 cm^3 (b) 7 cm
7. 8 cm

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Further Questions

1. (i) 3^9 (ii) 12 (iii) $3^4 \times 5^2 \times 7$

2. (i)
$$
\begin{aligned}
2\,744 &= 2 \times 1\,372 \\
&= 2 \times 2 \times 686 \\
&= 2 \times 2 \times 2 \times 343 \\
&= 2 \times 2 \times 2 \times 7 \times 7 \times 7 \\
\therefore \sqrt[3]{2744} &= \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} \\
&= 2 \times 7 \\
&= 14
\end{aligned}
$$

(ii)
$$
\begin{aligned}
21\,952 &= 2 \times 10\,976 \\
&= 2 \times 2 \times 5\,488 \\
&= 2 \times 2 \times 2 \times 2\,744 \\
&= 2 \times 2 \times 2 \times 2 \times 1\,372 \\
&= 2 \times 2 \times 2 \times 2 \times 2 \times 686 \\
&= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 343 \\
&= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7 \\
\therefore \sqrt[3]{21\,952} &= \sqrt[3]{(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (7 \times 7 \times 7)} \\
&= 2 \times 2 \times 7 \\
&= 28
\end{aligned}
$$

(iii)
$$
\begin{aligned}
91\,125 &= 3 \times 30\,375 \\
&= 3 \times 3 \times 10\,125 \\
&= 3 \times 3 \times 3 \times 3\,375 \\
&= 3 \times 3 \times 3 \times 3 \times 1\,125 \\
&= 3 \times 3 \times 3 \times 3 \times 3 \times 375 \\
&= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 125 \\
&= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \times (5 \times 5 \times 5) \\
\therefore \sqrt[3]{91125} &= \sqrt{(3 \times 3 \times 3) \times (3 \times 3 \times 3) \times (5 \times 5 \times 5)} \\
&= 3 \times 3 \times 5 \\
&= 45
\end{aligned}
$$

Chapter Two

RECIPROCALS

The concept of reciprocals is not entirely new to the learner as he/she has met it in division involving fractions. The learner's experience in reading squares, square roots and cubes is useful in finding reciprocals.

Objectives

By the end of the topic, the learner should be able to:

(i) find reciprocals of numbers by division.

(ii) find reciprocals of numbers using tables.

(iii) use reciprocals of numbers in computation.

Time: Seven lessons.

Teaching/Learning Activities

Reciprocal of Numbers by Division

- The teacher should define the word reciprocal, giving simple examples as in the students' book.
- The learner should be guided to find reciprocals of numbers by division, as in example 1.
- The learner should be given exercise 2.1.

Reciprocals of Numbers using Mathematical Tables

- The learner should be led to find reciprocals using mathematical tables, as in examples 2 and 3.
- The learner should be given question 1 of exercise 2.2.

Computation using Reciprocals

- The learner should be guided to solve problems involving reciprocals.
- The learner should be given questions 2 and 3 of exercise 2.2.

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Additional Hints

(i) The teacher should emphasise the use of reciprocal tables in solving problems involving divisions, such as $\frac{10}{4.356}$, $\frac{10}{0.2893}$, etc.

(ii) The learner should be given sufficient practice in finding the reciprocals of numbers greater than 10 and numbers less than 1 but greater than 0.

Evaluation

- Give a written test on the topic of reciprocals. Go through the test with the learners after marking and ensure all areas are understood.
- Present figures on the board and randomly ask the learners to find their reciprocals for further practice. Ensure all misconceptions are cleared.

Answers

Exercise 2.1

1. (a) $\frac{1}{3}$ (b) $\frac{1}{8}$ (c) $\frac{1}{12}$ (d) $\frac{1}{28}$ (e) $\frac{1}{256}$
2. (a) 0.4 (b) 0.2632 (c) 0.2381 (d) 0.1163 (e) 0.1064

Exercise 2.2

1. (a) 0.0178 (b) 0.009766 (c) 4.274
   (d) 21.83 (e) 0.0002289 (f) 335.8
2. (a) 80.08 (b) 8.242 (c) 1444.4
   (d) 10.54
3. (a) 2.013 (b) 1.0102 (c) 10.80
   (d) 172.7 (e) 3.684 (f) 2427.7
   (g) 1.569 (h) 9.4145 (i) 0.3077

Chapter Three

INDICES AND LOGARITHMS

The learner may have met indices (powers) when finding prime factors of composite numbers. Exercises should be given for practice on work pertaining to index notation. Logarithm is a new topic and should be introduced as in the students' book.

Logarithm tables should be available and used as much as possible for computation.

Objectives

By the end of the topic, the learner should be able to:

(i) define indices (powers).

(ii) state the laws of indices.

(iii) apply the laws of indices.

(iv) relate the powers of 10 to common logarithms.

(v) use the tables of common logarithms and antilogarithms in computation.

Time: Eighteen lessons.

Teaching/Learning Activities

Laws of Indices

- The teacher should define the word index as in the students' book. The learner should be guided to derive the laws of indices.
- The teacher should introduce negative, fractional and zero indices. To help the learner appreciate the meaning of negative and zero index, the teacher should use the laws of addition and subtraction as illustrated in the students' book.
- The learner should then be given exercise 3.1.

Powers of Ten and Common Logarithms

- The teacher should introduce the idea of logarithm as explained in the students' book. The learner should be led through examples 5 and 6, followed by exercise 3.2.
- After introducing the common logarithms, the teacher should then specifically discuss the logarithm of numbers:

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(i) between 1 and 10.
(ii) greater than 10.
(iii) between zero and one.

The teacher should introduce the term 'characteristic' and 'mantissa' at this stage. The learner should also be guided on how to find the antilogarithm of numbers as explained in the students' book.

Use of Logarithms and Antilogarithm in Computation

- Under this objective, the aim is to introduce the learner to computational skills using logarithm and antilogarithm tables. The learner should therefore be involved in solving problems using logarithm and antilogarithm tables, as in the students' book.
- The learner should be led through examples 11, 12 and 13, which should be followed by exercises 3.4 and 3.5.
- In finding roots of numbers, the learner should be involved in using logarithm tables as illustrated in the students' book.
- The teacher should stress the idea of making the characteristics to be divisible by a negative number as shown in the students' book.
- The learner should be given exercise 3.5 for practice.

Additional Hints

(i) The teacher should emphasise on proper arrangement of work.
(ii) As a concluding remark, the teacher should make the learner appreciate that the use of logarithm is a way of performing multiplication and division through addition and subtraction respectively.

Evaluation

- Give a written test on indices and logarithms. Go through the paper with the learners after marking to ensure all concepts are understood.
- Randomly present multiplication and division questions on the board and ask the learners to use logarithm tables to solve them. Test them similarly on indices.

Further Questions

1. $\sqrt{\left(\dfrac{3.684 \times 48.59}{5}\right)}$

2. $33.4 \div (49.34 \times 0.592)^{\frac{1}{2}}$

3. $(0.09)^{10}$

4. $\sqrt[4]{\left(\dfrac{0.418 \times 0.032}{(0.92)^2 \times 297}\right)}$

Answers

Exercise 3.1

1.	(a) 3	(b) $\frac{1}{5}$	(c) $\frac{1}{6}$	(d) 1	(e) 9
	(f) 216	(g) $\frac{1}{32}$	(h) $\frac{1}{3}$	(i) 1 024	(j) 10^9
	(k) 4 096	(l) (64)	(m) 9	(n) $\frac{1}{49}$	(p) 10^5
	(q) $\frac{1}{9^3}$	(r) 36	(s) $\left(3^{10}\right)^{\frac{1}{3}}$	(t) $\frac{1}{512}$	(u) $\frac{1}{2}$
2.	(a) 3^7	(b) 5^{-1}	(c) 2^4	(d) 3	(e) $\frac{6^5}{7^2}$
	(f) 5^{-4}	(g) $\frac{8}{7}$	(h) 9	(i) 2	(j) $\left(\frac{3}{4}\right)^5$
	(k) 1	(l) 2^{-4}	(m) 20^{-7}	(n) $2^5 \times 5^{\frac{1}{2}}$	
3.	(a) x^6	(b) y^9	(c) n^6	(d) a^{12}	(e) a^2
	(f) $(2n)^8$	(g) $\frac{x^{16}}{n^{12}}$	(h) a	(i) a^6b^2	(j) x^2b^{13}
	(k) b^3n^5	(l) $a^{12}b^3c^2$	(m) $a^{-10}b^{-2}$	(n) r^2	(o) xy^{-2}
	(p) $(xyt)^{n-2}$	(q) $a^{2n+2m} = a^{2(n+m)}$		(r) 1	(m) x^3y
	(n) $a^9b^{-2}c^4$				
4.	(a) $\frac{25}{81}y^2k^{-16}$	(b) $3a^2b^2$	(c) $(3^5ab^2)^{-1} = \frac{1}{3^5ab^2}$		(d) $(abc)^a$
	(e) $a^{-(2mn+4m)}$	(f) $r^{11}s^{18}t^{-15} = \frac{r^{11}s^{11}}{t^{15}}$		(g) $\left(\frac{6^{-10}a^{-1}b^2)}{5}\right)^{\frac{1}{3}}$	
	(h) $\frac{a^{\frac{21}{4}}}{120}$ or $\frac{1}{120a^{\frac{-21}{4}}}$		(i) $3x^{\frac{1}{3}}y^{\frac{5}{3}}$	(j) $\frac{2}{yk}$	
5.	(a) 2	(b) -2	(c) $\frac{1}{2}$	(d) 2	(e) $\frac{3}{2}$

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(f) $\frac{9}{8}$ (g) $\frac{3}{2}$ (h) $\frac{72}{11}$ (i) $\frac{7}{6}$ (j) 2

(k) 12 (l) 4 (m) 2 (n) 4 (p) 2^{-3y}

6. (a) (i) 2^3 (ii) 2^2 (iii) 2^1 (iv) 2^0

(v) 2^{-1} (vi) 2^{-2} (vii) 2^{-3}

(b) (i) 3^3 (ii) 3^2 (iii) 3^1 (iv) 3^0

(v) 3^{-1} (vi) 3^{-2} (vii) 3^{-3}

(c) (i) 10^3 (ii) 10^2 (iii) 10^1 (iv) 10^0

(v) 10^{-1} (vi) 10^{-2} (vii) 10^{-3}

Exercise 3.2

1. (a) $\text{Log}_3 9 = 2$ (b) $\text{Log}_2 16 = 4$ (c) $\text{Log}_3 27 = 3$

(d) $\text{Log}_2 32 = 5$ (e) $\text{Log}_3 81 = 4$ (f) $\text{Log}_5 125 = 3$

(g) $\text{Log}_{10} 1 = 0$ (h) $\text{Log}_2 1\,024 = 10$ (i) $\text{Log}_a b = n$

2. (a) $2^3 = 8$ (b) $4^2 = 16$ (c) $5^3 = 125$

(d) $10^x = 8$ (e) $b^c = 9$ (f) $3^3 = 27$

(g) $6^3 = 216$ (h) $x^y = 40$ (i) $4^y = 6$

(j) $y^2 = x$ (k) $10^4 = 10\,000$ (l) $2^4 = 16$

Exercise 3.3

1. (a) $10^{0.9138}$ (b) $10^{0.1673}$ (c) $10^{0.6749}$ (d) $10^{0.8603}$

(e) $10^{0.9926}$ (f) $10^{0.7538}$ (g) $10^{0.9105}$ (h) $10^{0.4972}$

(i) $10^{0.4343}$ (j) $10^{0.5228}$ (k) $10^{1.0899}$ (l) $10^{1.7760}$

(m) $19^{1.9186}$ (n) $10^{1.8573}$ (p) $10^{1.9827}$ (q) $10^{2.6350}$

(r) $10^{3.8989}$ (s) $10^{3.0107}$ (t) $10^{3.2817}$ (u) $10^{3.6935}$

(v) $10^{2.4373}$ (w) $10^{5.6767}$ (x) $10^{6.5922}$ (y) $10^{5.9815}$

(z) $10^{2.8334}$

Exercise 3.4

1. (a) 9.212×10^4 (b) 1.290×10^7 (c) 7.774×10^3

(d) 1.457×10^3 (e) 1.088×10^5 (f) 2.531×10^5

(g) 2.029×10^{10} (h) 5.684×10^4 (i) 8.162×10^6

(j) 1.698×10^9

2. (a) 2 (b) 3.830 (c) 1.710 (d) 6.125

(e) 7.991 (f) 5.005 (g) 1.156 (h) 13.596

(i) 2.145 (j) 2.985

3. (a) 5.327×10^2 (b) 2.426×10^5 (c) 4.623×10^3

(d) 6.724×10^2 (e) 9.376 (f) 2.25
(g) 1.309 (h) 1.339 (i) 1.801
(j) 2.282×10^2

4. (a) 3.722 (b) 2.461 (c) 3.056
(d) 5.4161 (e) 2.749 (f) 1.171
(g) 1.396×10 (h) 9.005 (i) 3.321×10^3
(j) 7.828×10

5. (a) $\bar{1}.1732$ (b) $\bar{1}.4538$ (c) 1.5465
(d) $\bar{1}.6321$ (e) $\bar{2}.8414$ (f) $\bar{2}.6857$
(g) $\bar{2}.3784$ (h) $\bar{4}.5752$ (i) $\bar{5}.8943$
(j) $\bar{5}.7185$

6. (a) 0.1282 (b) 0.1770 (c) 0.0247
(d) 0.0347 (e) 0.00447 (f) 0.000783
(g) 0.00944 (h) 0.0368 (i) 0.0000284
(j) 0.00481

7. (a) $\bar{3}.57$ (b) $\bar{6}.20$ (c) $\bar{6}.64$
(f) 6.54
(d) $\bar{1}.55$ (e) $\bar{3}.58$

8. (a) 3.070 (b) 0.02888 (e) 0.1071
(d) 0.1045 (e) 0.00542 (f) 12.05
(g) 0.00866 (h) 0.1282 (i) 0.0590
(j) 0.8017

Exercise 3.5

1. (a) (a) 0.4469 (b) 0.9312 (c) $\bar{1}.4877$
(d) $\bar{1}.8538$ (e) $\bar{1}.9269$ (f) $\bar{1}.175$
(g) $\bar{1}.0822$ (h) $\bar{2}.7464$ (i) $\bar{2}.8473$
(j) $\bar{2}.3969$

(b) (a) 0.2978 (b) 0.6208 (c) $\bar{1}.6585$
(d) $\bar{1}.9025$ (e) $\bar{1}.9513$ (f) $\bar{1}.450$
(g) $\bar{1}.3881$ (h) $\bar{1}.1643$ (i) $\bar{1}.2315$
(j) $\bar{2}.9313$

(c) (a) 0.2235 (b) 0.4656 (c) $\bar{1}.7439$
(d) $\bar{1}.9269$ (e) $\bar{1}.9635$ (f) $\bar{1}.5876$

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(g) $\bar{1}.5411$ (h) $\bar{1}.3732$ (i) $\bar{1}.4237$
(j) $\bar{1}.1985$

2. (a) (a) 21.863 (b) 49.608 (c) 59.766
(d) 68.433 (e) 0.186 (f) 0.0676
(g) 0.082 (h) 0.027 (i) 8.834
(j) 19.013

(b) (a) 7.819 (b) 13.501 (c) 15.28
(d) 16.730 (e) 0.326 (f) 0.166
(g) 0.189 (h) 0.090 (i) 4.273
(j) 7.124

3. (a) 1.352 (b) 0.00555 (c) 332.0
(d) 0.021 (e) 31.75 (f) 0.812
(g) 6.027

Further Questions

1.

No.	S. form	Log
3.684	3.684×10^{0}	0.5663
48.59	4.859×10^{1}	1.6865 + 2.2528
5		0.6990 – 1.5538 ÷ 2
5.983	5.983×10^{0}	= 0.7769

2.

No.	S. form	Log
33.4	3.34×10^{1}	1.5237
49.34	4.934×10^{1}	1.6932
0.592	5.92×10^{-1}	$\bar{1}.7723$
		1.4655 ÷ 2
		0.73275 = 0.7328
6.179	6.179×10^{0}	0.7909

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(c)

No.	S. form	Log
0.09	9.0×10^{-2}	$\bar{2}.9542 \times 10$
		$= -20.0000 + 9.5420$
		$= -11 + 0.5420$
0.00000000003483	3.483×10^{-11}	$= \overline{11}.5420$

(d)

No	S. form	Log
0.418	4.18×10^{-1}	$\bar{1}.6212$
0.032	3.2×10^{-2}	$\underline{\bar{2}.5051}$ +
	$\underline{\bar{2}.1263}$	$\bar{2}.1263$
0.92	$(9.2 \times 10^{-1})^{-2}$	$\bar{1}.9638 \times 2$
		$= \bar{1}.9276$
297	2.97×10^{2}	$\underline{2.\ 4728}$ +
	$\underline{2.\ 4004}$	$\underline{2.4004}$ –
		$\bar{5}.7259 \times \frac{1}{4}$
		$= \frac{-5 + -3 + 0.7259}{4} + 3$
		$= \frac{-8 + 3.7259}{4}$
		$= -2 + 0.931475$
0.08541	$10^{-2} \times 8.541$	$= \bar{2}.9315$

Chapter Four

EQUATIONS OF STRAIGHT LINES

The learner has met co-ordinates and graphs, which is pre-requisite knowledge for this topic. The topic exposes the learner to the concept of gradient and its relationship with equations of straight lines.

Objectives

By the end of the topic the learner should be able to:

(i) define gradient of a straight line.

(ii) determine gradient of a straight line using known points.

(iii) determine the equation of a straight line using gradient and one known point.

(iv) express a straight line equation in the form $y = mx + c$.

(v) interpret the equation $y = mx + c$.

(vi) find the x and y intercepts from the equation of a line.

(vii) draw the graph of a straight line using gradient, x and y-intercepts.

(viii) state the relationship between gradients of perpendicular lines.

(ix) state the relationship between gradients of parallel lines.

(x) apply the relationship of gradients of perpendicular and parallel lines to get equations of straight lines.

Time: Twelve lessons.

Teaching/Learning Activities

Definition and Determination of Gradient of a Line through Known Points

- The teacher should discuss with the learner gradient as in the students' book.
- The learner should be guided through examples 1, 2 and 3.
- The learner should be given exercise 4.1.

The Equation of a Straight Line

- The teacher should discuss with the learner examples 4 and 5 in the students' book.
- The learner should be given questions 1 and 2 of exercise 4.2.

Expression of a Straight Line Equation in the Form $y = mx + c$ and its Interpretation

- The teacher should lead the learner to understand the expression of a linear equation in the form $y = mx + c$, as in the students' book.
- The learner should be led through example 6.
- The learner should be given questions 3 and 4 of exercise 4.2.

The x and y Intercept of a Line

- The teacher should discuss the y and x intercept as in the students' book and guide the learner through example 7.
- The student should be given the remaining problems, i.e, questions 5, 6 and 7 of exercise 4.2.

Graph of a Straight Line using Gradient, x and y-Intercept

- The learner should be led through drawing a graph using intercepts as illustrated in figure 4.10.
- The learner should be given exercise 4.3.

Relationship between Gradients of Perpendicular Lines

- The learner should be involved in identifying perpendicular lines as in figure 4.11.
- The teacher should lead the learner to establish the relationship $m_1 \times m_2 = -1$, as in the students' book.
- The learner should be guided through example 8.
- The learner should be given exercise 4.4.

Relationship of Gradients of Parallel Lines

- The learner should draw lines of the same gradient and comment on the relationship between the lines, as in the students' book.
- The teacher should discuss the gradients of the lines as in figure 4.12.
- The learner should be given exercise 4.5.

Additional Hint

An alternative method of finding the equation of a line is by substituting known values in the equation $y = mx + c$. For example, consider a line whose gradient is 3 and passes through (1, 5). Its equation can be found as shown;

$y = mx + c$

$m = 3$

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Therefore, $y = 3x + c$
Substituting (1, 5) in the equation;
$5 = 3(1) + c$
$2 = c$
Thus, the equation of the line is $y = 3x + 2$

Evaluation

- Give a written test on equations of straight lines. Go through the test with the learners after marking to ensure they grasp the topic well.
- Give random questions on the topic on the chalkboard and ask the learners to solve them. Through discussion, ensure learners understand the concepts well.

Answers

Exercise 4.1

1. (a) L_2, L_3, L_4 (b) L_1, L_8 (c) L_5, L_6 (d) L_1
2.

	Change in x-coordinate	Change in y-coordinate	Gradient
(a)	3	3	1
(b)	7	10	$\frac{10}{7}$
(c)	7	–5	$\frac{-5}{7}$
(d)	2	0	0
(e)	4	–6	$\frac{-3}{2}$
(f)	1	4	4
(g)	–12	–1	$\frac{1}{12}$
(h)	0	8	undefined

3. $L_1 = \frac{1}{2}$, $L_2 = \frac{1}{3}$, $L_1 = \frac{-2}{3}$
4. (a) $\frac{-4}{3}$ (b) $\frac{-1}{3}$ (c) –1 (d) 0 (e) –1 (f) $\frac{10}{7}$
5. (a) $\frac{1}{2}$ (b) $\frac{4}{3}$ (c) –2 (d) –2 (e) $\frac{-3}{2}$ (f) 0
6. (a) Any point on the line $y = 5x - 3$.
   (b) Any point on the line $y = \frac{-3}{2} + 4$
   (c) Any point on the line $y = 5$
   (d) Any point on the line $y = \frac{x}{3} - 1$
   (e) Any point on the line $y = \frac{x}{2} + \frac{1}{4}$
   (f) Any point on the line $y = \frac{-3}{4}x + 3$

7. (a) (i) Draw the lines $y = x + c$ with two different values of c.
(ii) Any two points on the line.
(b) (i) Draw the line $y = 4x + c$ (where c is any rational number).
(ii) Any two points with any two different values of c.
(c) (i) Draw the line $y = -2x + c$ (where c is any rational number).
(ii) Any two points with any two different values of c.
(d) (i) Draw the line $y = 5x + c$ (where c is any rational number).
(ii) Any two points with any two different values of c.
(e) (i) Draw the line $y = \frac{1}{2}x + c$ (where c is any rational number).
(ii) Any two points with any two different values of c.
(f) (i) Draw the line $y = 4x + c$ (where c is any rational number).
(ii) Any two points with any two different values of c.
(g) (i) Draw any horizontal line.
(ii) Any two points on different horizontal lines.
(h) (i) Draw any vertical line.
(ii) Any two points on different vertical lines.

Exercise 4.2

1. (a) $\frac{7}{3}, 0$ (b) $3, 0.5$ (c) $\frac{-1}{2}, \frac{7}{4}$ (d) $0, \frac{7}{3}$
(e) $\frac{3}{2}, -2$ (f) $\frac{6}{5}, \frac{-3}{5}$ (g) $-3, -7$ (h) $\frac{5}{3}, 2$
(i) $\frac{4}{9}, 10$ (j) $-1, 2$ (k) $\frac{-5}{6}, \frac{-5}{12}$ (l) $-20, -60$ (m) $\frac{-a}{b}, \frac{-c}{b}$
2. (a) $y = 4x - 3$ (b) $4y = 3x + 15$ (c) $y = 2x + 16$
(d) $y = -\frac{1}{3}x + 4$ (e) $y = -5$ (g) $y = mx + (2 - m)$ or $y = mx - (m - 2)$
(h) $y = mx + (b - ma)$
4. (a) $y = 3x$ (b) $y = 6x - 4$ (c) $y = 6x + 4$
(d) $y = \frac{-3}{2}x + \frac{3}{2}$ (e) $y = 7$ (f) $y = -x + 6$
(g) $y = x - 10$ (h) $y = \frac{1}{3}x - \frac{5}{6}$ (j) $y = \dfrac{(d - b)(x - a)}{c - a} + b$
(k) $y = \dfrac{(y_2 - y_1)(x - x_1)}{x_2 - x_1} + y_1$
5. (a) $(\frac{3}{7}, 0)$ (b) $\frac{-2}{3}, 0)$ (c) $(-12, 0)$
(d) $(\frac{5}{8}, 0)$ (e) $\left(\frac{-c}{m}, 0\right)$ (f) $\left(\frac{-c}{a}, 0\right)$
6. (a) $y = -x - 2$ (b) $y = \frac{4}{3}x + 4$ (c) $y = \frac{1}{5}x - 1$

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(d) $y = \frac{-4}{3}x + 4$ (e) $y = \frac{-bx}{a} + b$

7. C(3, –2), B(–5, –2), $y = 2x + 8$

Exercise 4.3

Check for accuracy of graphs.

Exercise 4.4

1. Perpendicular lines

   (a), (b), (d), (f)

2. (a) $y = \frac{-1}{5}x + \frac{13}{5}$ (b) $y = \frac{1}{7}x - \frac{25}{7}$ (c) $y = \frac{1}{3}x - \frac{5}{6}$

   (d) $y = x + 6$ (e) $x = -2$ (f) $y = \frac{-1}{m}x + \left(\frac{bm + a}{m}\right)$

3. (a) $y = \frac{-1}{2}x - \frac{3}{2}$ (b) $y = 2x + 1$

4. Diagonal AC, $y = 3x - 1$

   BD, $y = \frac{-1}{3}x + \frac{22}{3}$

   Co-ordinates of D (1, 7)

5. DA: $y + 3x = -1$, DB: $y = \frac{-1}{2}x + 4$ or $2y = -x + 8$

   DC: $y = \frac{1}{3}x + \frac{17}{3}$ or $3y = x + 17$

6. B(3, 4), C(–5, 0), D(–3, –4)

Exercise 4.5

1. Parallel lines

   (a), (c), (e)

2. (a) $y = \frac{-2x}{7}$ (b) $y = -x + 3.5$ (c) $y = \frac{2x}{3} - \frac{4}{3}$

   (d) $y = \frac{3x}{7} + \frac{44}{7}$ (e) $y = 4x + \frac{16}{3}$ (f) $y = -2x - 3$

   (g) $y = \frac{-15}{22}x + 1$ (h) $x = -3$ (i) $y = 3$

3. $y = 5$, $y = x - 9$

4. (a) AD: $y = -3x - 8$

   BC: $y = -3x + 12$

   CD: $y = \frac{1}{3}x + \frac{16}{3}$

   (b) A(–3, 1), B(3, 3), C(2, 6), D(–4, 4)

   (c) AC, $y = x + 4$

   BD, $y = \frac{-1}{7}x + \frac{7}{2}$

Chapter Five

REFLECTION AND CONGRUENCE

This topic should strengthen what the learner already knows about the properties of reflection and help the learner to discover other properties.

Objectives

By the end of the topic, the learner should be able to:

(i) state the properties of reflection as a transformation.

(ii) use the properties of reflection in construction and identification of images and objects.

(iii) make geometrical deductions using reflection.

(iv) apply reflection in the cartesian plane.

(v) distinguish between direct and opposite congruence.

(vi) identify congruent triangles.

Time: Twelve lessons.

Teaching/Learning Activities

Lines and Planes of Symmetry

- The teacher should devise activities which will bring forth the idea of symmetry, as in the students' book.
- The learner should be given exercise 5.1.

Reflection

- The teacher should guide the learner in the activity involving figure 5.4 in the students' book.
- The teacher should lead the learner through example 1.
- The learner should be given exercise 5.2.

Geometrical Deductions using Reflection

- The teacher should guide the learner to deduce the following, as in the students' book:
  (i) Vertically opposite angles are equal.
  (ii) Base angles of an isosceles triangle are equal.
  (iii) Angle sum of a triangle is 180^0.
  (iv) Mirror line is a perpendicular bisector.
- The learner should be given exercise 5.3.

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Congruence

- The teacher should discuss congruence as in the students' book. Examples in real life should be included.
- The learner should be given exercise 5.4.

Congruent Triangles

- The teacher should discuss the four conditions for congruence in triangles, giving appropriate illustrations as in the students' book.
- The teacher should lead the learner to appreciate the ambiguous cases.
- The learner should be given exercise 5.5.

Additional Hints

The teacher should make the topic as practical as possible. Use of realia is highly encouraged.

Evaluation

- Give a written test on reflection and congruence. Go through the paper with the learners after marking to ensure all is clean.
- Present random questions pertaining to the topic to the learners. Through discussion, ensure all the questions can be answered adequately.

Answers

Exercise 5.1

1. (a) 1 (b) 5 (c) 6 (d) 4
   (e) none (f) none (g) 3 (h) 2
   (i) 8 (j) infinite
2. A, B, C, D, E, H, I, K,
   M, O, T, U, V, W, X, Y
3. (a) 4 (b) infinite (c) 3 (d) infinite
4. For example, right eye and left eye; left nostril and right nostril, right ear and left ear; left half of mouth and right half of mouth; right hand and left hand, and so on
5. 2

Exercise 5.2

1. Check for accuracy of drawings.
2. Check for correct construction. 33^0

3. Check for the angle bisector of 80^0.

4. (a) Check for correct construction.
   (b) Check for correct construction.
   (c) Check for correct construction.

5. (a) $A'(4.5, -1.5)$ $B'(7.5, -3)$ $C'(5, -3.5)$ $D'(4.5, -2.5)$
   (b) $A''(1.5, 4.5)$ $B''(3, 7)$ $C''(3.5, 5)$ $D''(2.5, 4.5)$
   (c) $A'''(-4.5, 1.5)$ $B'''(-7.5, 3)$ $C'''(-5, 3.5)$ $D'''(-4.5, 2.5)$
   (d) $A''''(-1.5, -4.5)$ $B''''(-3, -7.5)$ $C''''(-3.5, -5)$ $D''''(-2.5, -4.5)$

6. (a) $A'(2, 12)$ $B'(2, 7)$ $C'(7, 7)$
   (b) $A'(2, 2)$ $B'(2, -3)$ $C'(7, -3)$
   (c) $A'(5, -7)$ $B'(5, -2)$ $C'(0, -2)$
   (d) $A'(-9, -7)$ $B'(-9, -2)$ $C'(-14, -2)$

7. (a) $A''(2, -5)$ $B''(5, -2)$ $C''(5, -12)$
   (b) $A''(2, -5)$ $B''(5, -2)$ $C''(5, -12)$

8. (a) $A''(4, -6)$ $B''(3, -2)$ $C''(7, -1)$
   $D''(7, -4)$ $E''(8, -5)$ $F''(7, -6)$
   (b) $A'''(4, 6)$ $B'''(3, 2)$ $C'''(7, 1)$
   $D'''(7, 4)$ $E'''(8, 5)$ $F'''(7, 6)$

9. Final image is a circle, centre $(-2, 4)$, radius 3 units.

10. $(-2, 5)$, $(-1, 6)$, $(-2, 7)$, $(-1, 8)$, $(-2, 9)$

11. The vertices of the image are $(3, -8)$, $(4, -8)$, $(4, -6)$, $(5, -8)$

12. The vertices of the image are $(-2, -2)$, $(-3, -2)$, $(-3, -4)$ and $(-4, -4)$

13. (a) $(-12, 4)$, $(-13, 2)$, $(-14, 4)$
    (b) $(4, -12)$, $(2, -13)$, $(4, -14)$

14. Check for accurate drawing.

15. (a) $(4, 2)$, $(1, 1)$, $(-1, 2)$
    $(2, 4)$, $(2, 7)$, $(4, 6)$
    (b) $(-2, -4)$, $(-1, -1)$, $(-2, 1)$,
    $(-4, -2)$ $(-7, -2)$ $(-6, -4)$

16. $A'(12, 6)$, $B'(12, 4)$, $C'(10, 6)$, $D'(10, 8)$
    $E'(8, 8)$, $F'(10, 10)$, $G'(12, 10)$, $H'(12, 12)$
    $J'(14, 10)$, $K'(14, 8)$, $L'(16, 8)$, $M'(14, 6)$

17. $A'(4, -4)$ $B'(-6, -6)$ $C'(-4, -6)$ $D'(-2, -6)$

18. Co-ordinates of the:
    (a) first image are $(-9, 2)$, $(-10, 2)$, $(-11, 4)$ $(-13, 3)$, $(-13, 6)$, $(-9, 8)$, $(-9, 6)$, $(-12, 6)$, $(-9, 5)$

(b) second image are $(2,-9)$, $(2,-10)$, $(4,-11)$, $(3,-13)$, $(6,-13)$, $(8,-9)$, $(6,-9)$, $(6,-12)$, $(5,-9)$

Exercise 5.3

1. $y = x + 2$, $y + x = 2$
2. (a) $y = 9$ (b) $x = 0$ (c) $x = -3$
   (d) $x = 2.5$ (e) $y = x$
3. (a) Isosceles (b) $y = -3$
4. $\left(\frac{-1}{2}, 1\right)$, $(0, 3)$, $(0, 4)$, $(-3, 4)$, $y = -x$
5. (a) Rhombus (b) $y = 3x$, $3y + x = 15$
6. (a) $AB = 2\sqrt{5}$ units (b) $AB = A'B'$
7. (a) Kite (b) $x = -3$
   (c) $P'(7,-3)$, $Q'(3,-5)$, $R'(1,-3)$ $S'(3,-1)$
   (d) $y = -3$
8. (a) $x = 4$ (b) $(4, 1)$
9. $x = 3$

Exercise 5.4

1. (a) (i) and (iii) are directly congruent, (ii) and (iv) are directly congruent
   (b) (i) and (ii), (i) and (iv), (ii) and (iii) are oppositely congruent
2. (a) Triangles MNO, SUT, KLJ are directly congruent
   (b) Triangle DFE is oppositely congruent to triangles MNO, SUT and KLJ.
3. Isosceles and equilateral triangles

Exercise 5.5

1. QR = RS, (given), PQ = PS (given), PS is a common side. Therefore, ΔPQR is congruent to ΔPSR (SSS).
2. AOC = $\angle AOB$ (given), OC = OB (radii of same circle). OA is common
   Therefore, ΔAOC is congruent to ΔAOB (SAS).
3. AB = DC (opposite sides of a parallelogram), $\angle BAE = \angle DCF$ (opposite angles of a parallelogram), AE = FC (BE is parallel to FD). Therefore, ΔABE is congruent to ΔCDF (SAS).
4. (a) AB = AC (given), $\angle BAD = \angle DAC$ (bisector of $\angle BAC$) or

AD is common
Therefore, ΔABD is congruent to ΔACD
Thus, BD = DC.

(b) AC = AB (given) $\angle ACD = \angle ABD$ (base angles of an isosceles triangle ABC).
BD = DC
Therefore, AD is perpendicular to BC.

5. PS = QR (given), PQ is common $\angle SPQ = \angle PQR$
($\angle SPX = \angle RQX$ since PS = PX = QX = QR. $\angle XPQ$ = XQP triangle PXQ is isosceles).

6 - 13. Check for correct constructions.

14 - 19. Check for correct showing.

Chapter 6

ROTATION

This is a new topic to the learner. The learner has, however, met construction of angles and perpendicular bisectors, ideas which are useful in the topic.

Objectives

By the end of the topic, the learner should be able to;

(i) state properties of a rotation as a transformation.
(ii) determine centre and angle of rotation.
(iii) apply properties of rotation in the cartesian plane.
(iv) identify point of rotational symmetry of solids.
(v) state order of rotational symmetry of solids.
(vi) deduce congruence from rotation.

Time: Twelve lessons.

Teaching/Learning Activities

Angle and Centre of Rotation and Applications

- The learner should be led to practically establish the properties of a rotation as in:
  (i) example 1 in the students' book when given an object and centre of rotation.
  (ii) sub-topic 6.2, given the image and the object in determination of centre and angle of rotation.
- The learner should be led to extend the same knowledge to the cartesian plane as illustrated in figure 6.7.
- The learner should be given exercise 6.1.

Rotational Symmetry and Congruence

- The learner should be led to practically deduce the order of rotational symmetry of plane figures, as in sub-topic 6.4 in the students' book.
- The learner should be given exercise 6.2.
- Rotational symmetry should be used to show that the object and image are directly congruent after a rotation.

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- The learner should be guided to identify axes of rotational symmetry of common solids and in stating the order of their rotational symmetry, as in sub-topic 6.5.
- The learner should be given exercise 6.3.

Additional Hints

(i) The teacher is encouraged to use worksheets and models.

(ii) Rotation should be described using both negative and positive angles.

(iii) Angles greater than 360^0 should also be considered.

Evaluation

- Give a written test on rotation. Discuss the paper with the learners after marking to ensure the concept is understood clearly.
- Draw different shapes on the chalkboard and ask the learners, in turn, to perform rotations on them, through given angles and about given centres. Through discussion, ensure the topic is clearly understood.

Answers

Exercise 6.1

1. (a) (0, 0), 180^0
   (b) (0, –3), (–2, –2), (0, 0)
2. Centre (6.5, 0) angle of rotation is 180^0; (11, 0)
3. (a) A′(5, –3), B′(5, –7), C′(0, –5), D′(0, –1)
   (b) A′(–3, –5), B′(–7, –5), C′(–5, 0), D′(–1, 0)
4. (i) 90^0 (ii) P″(5, 2)
5. A′(3.95, 2.05), B′(4.4, 1.15), C′(4.2, 0.5) D′(3.3, 2.2)
   Image (6, 1.5)
6. (i) (8, 2) (ii) (2,–4), (iii) (1, 5), (iv) (12, 8)
7. A(–2, 4), B(–6, 1), C(–2,1)
8. (a) (2, 1) (b) O″(–3, –1), P″(–3, 2) Q″(O, 2), R″(O, –1)
9. Negative quarter turn about N(–4, –3);
   Image of (3, –4) is (–5, –10)

Exercise 6.2

1. A – 1 B – 1, H – 2, I – 2, N – 2
   O – Infinite, S – 2, X – 2
2. (–1.5, 11), (–5. 5, 1). There are more possibilities
3. (a) 4 (b) 6 (c) 2 (d) 3
   (e) 2 (f) 1 (g) 3 (h) 1
4. Check for accuracy of drawings.

Exercise 6.3

Check correct drawings.

Chapter Seven

SIMILARITY AND ENLARGEMENT

The learner has known how to measure lengths and calculate area and volume of regular shapes. Ratios, construction of triangles and scale drawing have also been met.

These ideas are useful when dealing with similarity and enlargement.

Objectives

By the end of the topic, the learner should be able to:

(i) identify similar figures.
(ii) construct similar figures.
(iii) state and apply the properties of enlargement to construct objects and images.
(iv) apply enlargement in cartesian planes.
(v) state the relationship between linear, area and volume scale factors.
(vi) apply the scale factors to real life situations.

Time: Nineteen lessons.

Teaching/Learning Activities

Similar Figures

- The teacher should introduce the idea of similarity by discussing rectangles, squares, circles, triangles and other regular polygons which have the same shape and size. Also to be discussed are those with same shape but different sizes
- The teacher should discuss the properties of similar figures, as in the students' book.
- The learner should be guided through examples 1 and 2.
- The learner should be given exercise 7.1.

Enlargement

- The teacher should discuss enlargement as a transformation, as in the students' book.
- The learner should be guided through examples 3, 4, 5 and 6.
- The learner should be given exercise 7.2.

- The teacher should discuss negative scale factor using figure 7.25 in the students' book.
- The learner should do exercise 7.3.

Relationship between Linear Scale Factor, Area Scale Factor and Volume Scale Factor

- The teacher should guide the learner to realise the relationship between linear scale factor and area scale factor.
- The teacher should guide the learner through examples 7, 8 and 9.
- The learner should do exercise 7.4.
- The teacher should guide the learner to establish the relationship between linear scale factor and volume scale factor, as in the students' book.
- The learner should be guided through examples 10 and 11.
- The learner should do exercise 7.5.

Evaluation

- Give a written test on similarity and enlargement. Discuss the paper with the learners after marking to ensure understanding of all concepts.
- Present random questions on such areas as linear, area and volume scale factors. Let the learners answer them and discuss the same.
- Give the learners mixed exercise 1 to assess them on topics met so far. Discuss all questions after marking.

Further Questions

1. $A'(1, 4)$ is the image of $A(1, -2)$ under an enlargement scale factor $\frac{-1}{2}$. Calculate the centre of enlargement.

Answers

Exercise 7.1

1. a and e, b and j, c and n, d and m, f and h, g and p, i and d, k and q.
2. All are similar
3. All are similar
4. a and c
5. The following triangles are similar:
   ABC and ACH, CFG and CGH, CED and CEF

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ABC and CEF, ACH and CDE, ACH and AEG,
AEG and CEF

6. (a) AB and CD, AO and OD, OB and OC
   (b) $\angle COD$ and $\angle AOB$, $\angle CDO$ and $\angle BAO$, $\angle DCO$ and $\angle ABO$
   (c) They are similar.
7. BC = 7.5 cm
8. SR = 4.5 cm, PX = 7.2 cm
9. HG = 5.42 cm, FG = 2.08 cm
10. Construct ΔPQR such that $\angle PQR = 46^0$, $\angle PQ = 7.5$ cm and QR = 6 cm
11. Construct rectangle PQRS such that;
    $\angle P = \angle Q = \angle R = \angle S = 90^0$, PQ = 5 cm and QR = $11\frac{2}{3}$ cm
12. Construct rectangle YEFG such that FY = 6 cm is the diagonal, YE = 4.8 cm and EF = 3.6 cm.
13. QT = 20 cm

Exercise 7.2

1. (a) Check on correct construction.
   (b) Check on correct construction.
   (c) Check on correct construction.
2. Check on correct construction.
3. (a) Check on correct construction.
   (b) Check on correct construction.
   (c) Check on correct construction.
4. (a) Check on correct construction.
   (b) Check on correct construction.
5. Centre of enlargement is (–3, –3), scale factor 2.
6. (a) P′(1.5, 2), Q′(2, 2), R′(3, 2), S′(3.5, 0.5), T′(2.5, 0)
   (b) P′(9, 12), Q′(12, 12), R′(18, 12), S′(21, 3), T′(15, 0)

Exercise 7.3

1. (a) A′(–4, –2), B′(–9, –2), C′(–7, 2), D′(–2, 2)
   (b) A′(–8, –4), B′(–18, –4), C′(–14, 4), D′(–4, 4)
   (c) A′(–1, –0.5), B′(–2.25, –0.5), C′(–1.75, 0.5), D′(–0.5, 0.5)
2. (a) B is the centre, the linear scale factor is 2.
   (b) A is the centre, the linear scale factor is 3.
   (b) C is the centre, the linear scale factors is $\frac{1}{4}$

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3. 7.5 cm, 10 cm, 12.5 cm
4. 1: 200 000
5. (a) 2.25 cm by 3.75 cm
   (b) 4.5 cm by 7.5 cm
   (c) 9 cm by 15 cm
6. 38. 5 cm
7. Linear scale factor is 3 : 2 000. QR = 6 cm and PR = 4.5 cm.
8. 18.7 cm.

Exercise 7.4

1. 65 m^2
2. (a) 6.25 (b) 120 cm^2
3. (a) 20 cm by 20 cm (b) 844 tiles
4. (a) 50 m^2 (b) 2 : 5 (c) 4 m
5. (a) 9 : 49 (b) 196 cm^2
6. 1 : 9

Exercise 7.5

1. (a) 3 (b) 9
2. 1 : 8
3. 8
4. (a) 1 : 27 (b) 324 kg
5. 3.7 kg
6. 593 m*l*
7. 252 cm^2
8. 360 000 cm^3
9. 18 cm
10. (a) 2 : 5 (b) 4 : 25
11. (a) 8.16% (b) 12.5%
12. (a) 2 : 3 (b) 4 : 9
13. (a) 31.5 cm, 21 cm (b) 9 : 4 (c) 27 : 8

Further Questions

Let the centre of enlargement be O(x, y). Where k is the scale factor.
$k\overline{OA} = \overline{OA}$

Thus, $\frac{-1}{2}\begin{pmatrix}1 - x\\ -2 - y\end{pmatrix} = \begin{pmatrix}1 - x\\ 4 - y\end{pmatrix}$

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$$\begin{pmatrix}1-x\\-2-y\end{pmatrix} = -2\begin{pmatrix}1-x\\4-y\end{pmatrix}$$

$$\begin{pmatrix}1-x\\-2-y\end{pmatrix} = \begin{pmatrix}-2+2x\\-8+2y\end{pmatrix}$$

$1 - x = -2 + 2x$ $\quad$ $-2 - y = -8 + 2y$

$3 = 3x$ $\quad$ $6 = 3y$

$1 = x$ $\quad$ $2 = y$

The centre of enlargement is (1, 2).

Mixed Exercise 1

1. 0.329
2. (i) $\angle QPR = 72^0$ (ii) RQ = 37.5 cm PR = 33.75 cm
3. –3.75
4. y = 368
5. $y = \frac{5}{48}$
6. (a) l_1: $2y + x = 5$ (b) l_2: Gradient = $\frac{8}{3}$
   (c) l_2: $3y = 8x - 17$ (d) $\left(\frac{49}{19}, \frac{49}{19}\right)$
   (e) $2y + x = 14$ (f) $y = \frac{3}{8}x + 5$
7. (a) 7.093 (b) 7.327
8. 252 cm^2
9. 1.63
10. 4.718 cm
11. 500 ha
12. (a) $3y - 4x + 6 = 0$ (b) $10y + 4x - 1 = 0$
13. (a) 20 cm (b) 10 cm (c) 11.18 cm
14. a = 5, b = 1
15. 2.05×10^{-1}
16. 21 km
17. (a) P′(–3, 0); Q′(–7, –2); R′(–5, –8)
    (b) P′(2,0); Q′(3, 0.5); R′(2.5, 2)
18. $y = \frac{1}{3}x + 4$ (b) (1.2, 3.6)
19. 0.1374
20. 20 cm
21. (a) Scale factor 1, centre (5, 1) (b) C′(7, 4)

22. $3y + x = 30$ and $y - 3x + 24 = 0$
23. 4.204 cm
24. $-2, \frac{-5}{12}, \frac{-5}{10}$
25. (a) (i) $P'(1, 1)$, $Q'(2, 1)$, $R'(2, 2)$, $S'(1, 2)$
(ii) $P'(-1, -1)$, $Q'(-2, -1)$, $R'(-2, -2)$, $S'(-1, -2)$
(iii) $P'(3, 3)$, $Q'(6, 3)$, $R'(6, 6)$, $S'(3, 6)$
(iv) $P'(-3, -3)$, $Q'(-6, -3)$, $R'(-6, -6)$, $S'(-3, -6)$
(v) $P'(\frac{1}{2}, \frac{1}{2})$, $Q'(1, \frac{1}{2})$, $R'(1, 1)$, $S'(\frac{1}{2}, 1)$
(vi) $P'(-\frac{1}{2}, -\frac{1}{2})$, $Q'(-1 -\frac{1}{2})$, $R'(-1, -1)$, $S'(-\frac{1}{2}, -1)$
(b) (i) (1, 1) (ii) +5
26. $A'''(-1, 3)$ $B'''(-3, 2)$ $C'''(-4, 4)$ $D'''(-3, 6)$
27. (a) $m_1 : y = x + 7$, $m_2 : y = -x - 1$
(b) Show the correct proof.
28. $x = 6$
29. (a) $P'(8, 0)$, $Q'(3, -3)$, $R'(1, 0)$, $S'(3, 3)$
(b) $P'(0, -8)$, $Q'(-3, -3)$, $R'(0, -1)$, $S'(3, -3)$
30. (a) Kite (b) $x = 4$ (c) right angle
31. Show correct deduction.
32. Show correct deduction.
33. (a) Centre (3, 0), angle 180^0
(b) (6, 4) and (7, –2)
34. Check correct deduction.
35. (4, –4) and (4, –8)
36. Check for correct proof.
37. Check for correct proof.
38. 5.926 cm^2
39. 0.003212
40. 12
41. (a) 2 (b) 10

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Chapter 8

PYTHAGORAS' THEOREM

The learner has met situations where Pythagoras' theorem is applied. In this topic, similarity is used to derive the theorem. The theorem is then applied in solving problems.

Objectives

By the end of the topic, the learner should be able to:

(i) derive Pythagoras' theorem.

(ii) solve problems including Pythagoras' theorem.

(iii) apply Pythagoras' theorem to real life situations.

Time: Four lessons.

Teaching/Learning Activities

Derivation of Pythagoras' Theorem

- The learner should be led through the derivation of Pythagoras' theorem, as in example 8.1 in the students' book.

Application of Pythagoras' Theorem

- The learner should be led to apply Pythagoras' theorem as in example 8.1 in the students' book. This should be extended to problems in real life situations.
- The learner should be given exercise 8.1.

Additional Hints

Pythagoras' theorem can also be derived as follows;

(i) Triangle ABC is right-angled at C, see figure 8.1. A perpendicular is dropped from C to the hypotenuse.

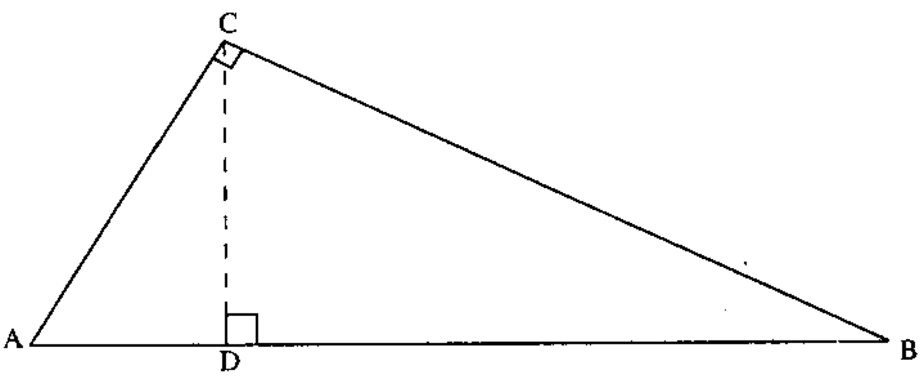

Fig. 8.1

The triangle BCD is similar to ΔBAC. Therefore;

$\frac{BC}{BA} = \frac{BD}{BC}$ and $\frac{AC}{BA} = \frac{AD}{AC}$

$(BC)^2 = BA.\ BD$ and $(AC)^2 = BA.\ AD$

This relationship can be used to derive the Pythagoras' theorem.

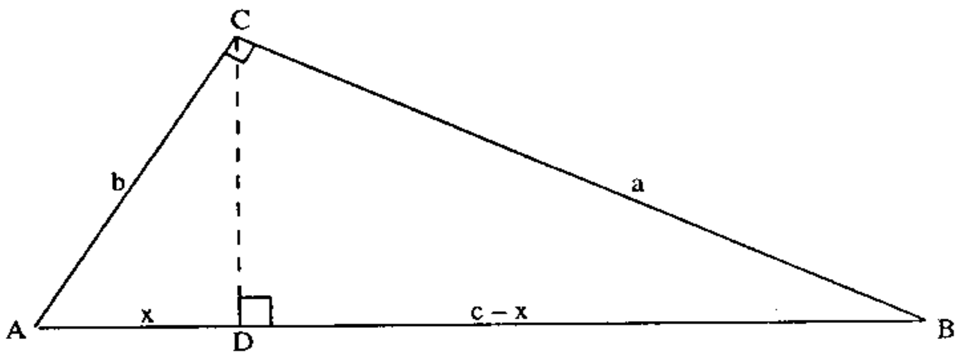

Fig. 8.2

In figure 8.2;

$b^2 = cx$

$a^2 = c(c - x)$

$a^2 + b^2 = cx + c(c - x)$

$a^2 + b^2 = c^2$

(ii) The learner should be led to establish the relationship between areas of squares drawn on the sides of a right-angled triangle. A grid may be used to enable the learner to use as many right angles as possible. The relationship should be used to enhance understanding of the theorem.

(iii) A table of Pythagorean triples should be given to the learner.

(iv) Knowledge or Pythagoras' theorem can be used to find the length of line segments in the cartesian plane.

Evaluation

- Give a written test on Pythagoras' theorem. Discuss the questions with the learners after marking.
- At random, present related questions to the learners on the chalkboard. Discuss the same after they attempt them to ensure the topic is grasped adequately.

Answers

Exercise 8.1

1. (a) 13 (b) 12 (c) 24 (d) 7.81
2. 10 cm
3. 5.66 cm
4. 10 cm
5. 4.243 cm
6. (a) 15 cm (b) 40 cm
7. 8.66 cm
8. 6 cm
9. 4 m
10. 8.95 m
11. 19.85 cm
12. 4.8 cm

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Chapter Nine

TRIGONOMETRIC RATIOS

This is a new topic to the learner. The three basic trigonometric ratios considered are sine, cosine and tangent. The topic is restricted to acute angles only.

Objectives

By the end of the topic, the learner should be able to:

(i) define sine, cosine and tangent using a right-angled triangle.
(ii) read and use tables of trigonometric ratios.
(iii) use sine, cosine and tangent in calculating lengths and angles.
(iv) establish and use the relationship of sine and cosine of complementary angles.
(v) relate the three trigonometric ratios.
(vi) determine the trigonometric ratios of special angles 30^0, 45^0, 60^0 and 90^0, without using tables.
(vii) read and use tables of logarithms of sine, cosine and tangent.
(viii) apply the knowledge of trigonometry to real life situations.

Time: Nineteen lessons.

Teaching/Learning Activities

Tangent, Sine and Cosine in a Right-angled Triangle

- The learner should be guided through the definitions of sine, cosine and tangent, as in the students' book.
- The learner should be given exercise 9.1.
- The learner should be guided through examples 1 and 2.
- The learner should be given exercise 9.2.

Tables of Trigonometric Ratios

- The learner should be assisted in identifying the tables to use when finding tangent, sine and cosine of angles.
- The learner should be guided through examples 3, 4, 5 and 6.
- The learner should be given exercise 9.3.
- The learner should be guided to find sines and cosines of angles from tables, as in examples 7 and 8.
- The learner should be given exercise 9.4.

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Sine and Cosine of Complementary Angles

- A table of the form shown below could be useful in assisting the learner to establish the relationship between sines and cosines of complementary angles.

Table 9.1

Angle	*Sine*	*Cosine*	*Complementary angle*	*Sine*	*Cosine*
30	0.50	0.87	60^0	0.87	0.50
45					
50					
60					
75					
90					

The learner should be guided to add values of sine and cosine from the trigonometric tables. The relationship that will be established is $\sin (90 - \theta)^0 = \cos \theta^0$

- The learner should be guided through example 9.
- The learner should be given exercise 9.5.

Trigonometric Ratios of Special Angles 30^0, 45^0, 60^0 and 90^0

- The teacher should lead the learner in finding the trigonometric ratios of special angles. Care should be taken not to mention surds.
- The learner should be given exercise 9.6.

Logarithms of Sine, Cosine, Tangent and Applications of Trigonometry to Real Life Situations

- The teacher should explain the reading and use of logarithms of trigonometric ratios.
- The learner should be guided through examples 10 and 11.
- The learner should be given exercise 9.7.

Additional Hints

(i) When reading the tables, it is advisable that students use a straight edge across the table at the appropriate line. This will reduce chances of taking the wrong value.

(ii) The relationship $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ could be derived as follows;

Consider triangle ABC below;

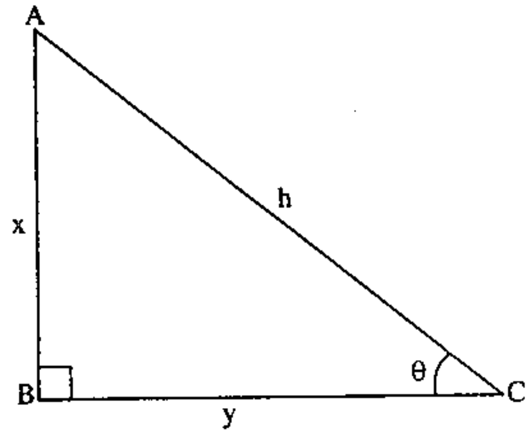

Fig. 8.3

$$\sin\theta = \frac{x}{h} \qquad\qquad \tan\theta = \frac{x}{y}$$

$$\therefore h\sin\theta = x \qquad\qquad = \frac{h\sin\theta}{h\cos\theta}$$

$$\cos\theta = \frac{y}{h} \qquad\qquad = \frac{\sin\theta}{\cos\theta}$$

$$\therefore h\cos\theta = y$$

(iii) There are two ways of finding logarithms of trigonometric ratios:

- By checking in the logarithms of sines, cosine and tangent tables.
- By getting the ratios first and then getting the logarithms of the ratios.

(iv) A mention of the relationship between the tangent of the angle a line makes with the horizontal and the gradient of the line should be made i.e., $\tan\theta = m$, where $y = mx + c$.

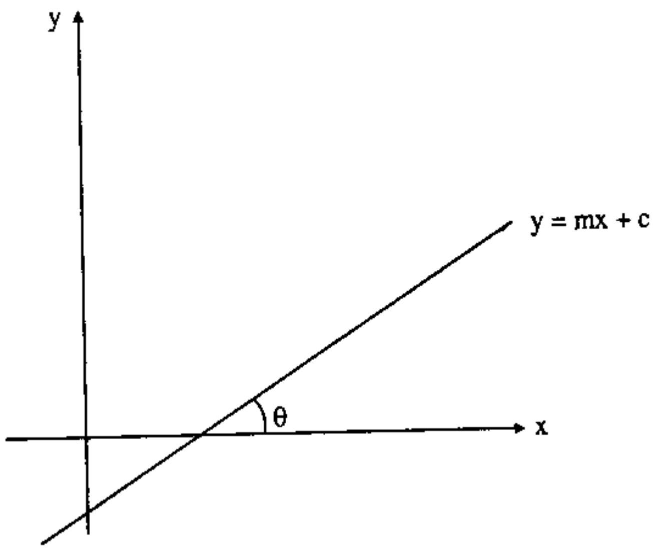

Fig. 8.3

(v) The product of tangent of an angle θ and the tan of its complement $(90 - \theta)$ is equal to one that is;
$\tan \theta^0 \tan (90 - \theta)^0 = 1$

Evaluation

- Give a written test on the trigonometry. Discuss the paper with the learners after marking.
- Give related random questions on sine, cosine and tangent on the chalkboard. Have them discussed after learners' attempt.

Answers

Exercise 9.1

1.

Angle	*Opposite Side*	*Adjacent Side*
θ_1	AB	BC
θ_2	BC	AB
α_1	DF	EF
α_2	EF	DF
β_1	GH	IH
β_2	IH	GH

2. $\tan \theta_1 = \frac{4}{3}$ $\tan \theta_2 = \frac{3}{4}$

$\tan \alpha_1 = \frac{3}{2}$ $\tan \alpha_2 = \frac{2}{3}$

$\tan \beta_1 = \frac{3}{4}$ $\tan \beta_2 = \frac{3}{4}$

3. (a) Check for correct constructions.

(b) (i) $\tan \theta = \frac{3}{4}$ $\tan \alpha = \frac{4}{3}$

(ii) $\tan \theta = \frac{3}{4}$ $\tan \alpha = 1$

(iii) $\tan \theta = \frac{3}{4}$ $\tan \alpha = \frac{4}{3}$

Exercise 9.2

1	(a) $15^0\ 18'$	(b) $25^0\ 45'$	(c) $30^0\ 30'$	(d) $34^0\ 45'$
2.	(a) 2.829	(b) 0.3391	(c) 1.407	
	(d) 1.980	(e) 6.410	(f) 0.1853	
	(g) 4.000	(h) 5 730	(i) 0.5673	
	(j) 26.43	(k) 4.759	(l) 1.138	
	(m) 9.514	(n) 0.9607	(p) 1.560	
3.	(a) 18.35^0	(b) 32.02^0	(c) 58.61^0	
	(d) 23.96^0	(e) 0.6990^0	(f) 41.34^0	
	(g) 10.58^0	(h) 30.55^0	(i) 78.91^0	
	(j) 45^0	(k) 15.85^0	(l) 63.45^0	
	(m) 74.76^0	(n) 36.50^0	(p) 88.60^0	
4.	36.87^0	5. 57.99^0	6. 21.0 m	
7.	12.02 m	8. 26.06^0, 63.94^0		

9. 19.6 cm and 17.09 cm or 39.99 cm and 24.09 cm

10. (a) 35.7 m (b) 34.57 m

11. 3.58^0 12. 115.5 km 13. $\angle ACB = 58^0$
$\angle BAC = 32.0^0$

14. AB = 59.3 km, BC = 125.84 km

15. x = 39.94 cm, b = 46.6 cm, c = 77.55 cm, y = 66.4 cm

16. p = 13 cm, x = 11.26 cm, y = 3.75 cm, q = 7.5 cm

Exercise 9.3

1. (a) $\cos \alpha_1 = \sin \alpha_2 = 0.8$; $\cos \alpha_2 = \sin \alpha_1 = 0.6$

(b) $\cos \theta_1 = \sin \theta_2 = \frac{12}{13}$; $\cos \theta_2 = \sin \theta_1 = \frac{5}{13}$

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(d) $\cos\beta_1 = \sin\beta_2 = 0.4472$; $\cos\beta_2 = \sin\beta_1 = 0.8944$

(e) $\cos w_1 = \sin w_2 = \frac{12}{15}$; $\cos w_2 = \sin w_1 = \frac{9}{15}$

(f) $\cos p_1 = \sin p_2 = \frac{1}{2}$; $\cos p_2 = \sin p_1 = \frac{\sqrt{3}}{2}$

(g) $\cos b_1 = \sin b_2 = 0.6476$; $\cos b_2 = \sin b_1 = \frac{16}{21}$

(h) $\cos\mu_1 = \sin\theta_2 = 0.5300$; $\cos\mu_2 = \sin\mu_1 = 0.8480$

2. (a) (i) 35.7^0 (ii) 30.0^0 (iii) 17.4^0
(iv) 36.9^0 (v) 44.4^0 (vi) 36.9^0
(vii) 34.8^0 (viii) 23.6^0 (ix) 56^0
(x) 25^0

(b) (i) 70.5^0 (ii) 38.9^0 (iii) 78.5^0
(iv) 53.1^0 (v) 67.4^0 (vi) 36.9^0
(vii) 53.9^0 (viii) 43.9^0 (ix) 76.1^0
(x) 56^0

3. (a) (i) $\cos\theta = \frac{4}{5}$ (ii) $\tan\theta = \frac{3}{4}$ (b) $\frac{\sqrt{3}}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{2}}{2}$

Exercise 9.4

1. (a) 19.68 (b) 35.95^0 (c) 2.99 (d) 58.21^0
(e) 10.50^0 (f) 79.55 (g) 30.08 (h) 4.58^0

2. (a) 80.54^0 (b) 43.98^0 (c) 4.44^0 (d) 30.0^0
(e) 66.36^0 (f) 18.54^0 (g) 90^0 (h) 42.52^0

3. (a) 0.5219 (b) 0.9765 (c) 0.7896 (d) 0.9172
(e) 0.1177 (f) 0.6445 (g) 0.4506 (h) 0.2261

4. (a) 0.1788 (b) 0.9119 (c) 0.9952 (d) 0.5000
(e) 0.0177 (f) 0.5596 (g) 0.8355 (h) 0.9518

5. (a) x = 13.57 cm, y = 8.48 cm
(b) y = 5.07 cm, h = 10.88 cm
(c) h = 15.32 cm x = 12.86 cm
(e) h = 4.57 cm, t = 2.22 cm

6. PR = 7.522 cm, QS = 11.81 cm

7. (a) 9.54 cm (b) 72.54^0

8. (a) 9.33 m (b) 22.1 m

9. 7.71 cm 10. 8.28 m 11. RS = 9.87 cm QR = 4.1 cm

12. a = 7.65 cm b = 10.39 cm c = 5.08 cm
d = 3.95 cm e = 9.19 cm

13. P = 5.9 cm q = 5.35 cm r = 4.93 cm s = 6.30 cm

14. 5.9 cm, 11.58 cm
15. 11.58 cm
16. 5.50 cm
17. (a) 63^0 (b) 13.23 cm (c) 11.78 cm
18. 15.68 cm
19. 20.34 cm
20. KL = 9.644 cm, KM = 7.453 cm

Exercise 9.5

1. $\frac{4}{5}$
2. $\cos\theta = \frac{4}{5}$ $\sin\theta = \frac{3}{5}$
3. 4.05^0
4. (a) sin A = 0.5, cos A = 0.866

Exercise 9.6

1. (a) $\frac{\sqrt{3}}{4}$ (b) $\sqrt{6}$ (c) $\frac{1+3\sqrt{3}}{2}$

   (d) $1\frac{1}{2}$ (e) 1 (f) 1
2. (a) $4\sqrt{7}$ (b) 82.82
3. (a) $20\sqrt{3}$ (b) 20 cm
4. (a) 6 cm (b) 3 cm
5. $x\frac{\sqrt{3}}{2}$
6. $13\sqrt{3}$ cm; $\frac{169}{4}\sqrt{3}$

Exercise 9.7

1. (a) 7.725 (b) 7.755 (c) 15.52 (d) 0.5446
2. (a) p = 5.439 r = 5.632 $\angle PQR = 44^0$
   (b) r = 10.53 $\angle PQR = 52.25^0$ $\angle PRQ = 37.75^0$
   (c) r = 3.036 q = 5.453 $\angle QPR = 33^0\,50'$
   (d) p = 3.798 q = 5.144 $\angle QRP = 42^0\,25'$
3. 4.91 m
4. (a) 71.5^0, 37^0 (b) 20.22 cm^2
5. (a) 51.31^0 (b) 77.38^0 (c) 8.20 cm^2
6. 21.34 cm^2
7. 8.00 cm
8. 6 km
9. North: 9.6 km
10. West: 18.9 km

AREA OF A TRIANGLE

The learner has already known how to find the area of a triangle, given the base and the height. In this topic, other formulae will be used to find the same.

Objectives

By the end of the topic, the learner should be able to:

(i) derive formula; Area = $\frac{1}{2}$ absin c.

(ii) solve problems involving area of triangles using the formula Area = $\frac{1}{2}$ absin c.

(iii) solve problems on area of a triangle using the formula; Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Time: Seven lessons.

Teaching/Learning Activities

Derivation and use of the Formula; Area = $\frac{1}{2}$ *absinc*

- The teacher should involve the learner in revision of trigonometric ratios, in particular, the sine of an angle.
- The teacher should discuss example 1 in the students' book. Show the learner how to apply the formula; Area = $\frac{1}{2}$ absinc by discussing example 2.

Area of a triangle using the formula Area = $\sqrt{s(s-a)(s-b)(s-c)}$

- The teacher should emphasise that this formula is applicable when all the three sides of the triangle are given.
- The teacher should discuss example 3 in the students' book.
- The learner should be given exercise 10.1.

Additional Hints

The area of a triangle given the length of the three sides may also be found through scale drawing. The perpendicular height (h) is then dropped from a vertex to the opposite side and the formula A = $\frac{1}{2}$bh used.

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Evaluation

- Give a written test on areas of triangles. Discuss the paper with the learners after marking.
- Present random questions to the learners for further practice and discussion

Answers

Exercise 10.1

1. (a) 26.83 cm^2 (b) 83.03 cm^2 (c) 152.8 cm^2
2. (a) 11.54 cm^2 (b) 6.677 cm^2 (c) 189.8 cm^2
3. (i) 30. 59 cm^2
   (ii) 42.8^0, 76.2^0 and 61^0
4. (a) 5.4 cm (b) 10.4 cm
5. 6.72 cm
6. 6 cm, 7 cm
7. 470.1 cm^2
8. 22.9 cm^2

Chapter Eleven

AREA OF QUADRILATERALS AND OTHER POLYGONS

This topic is not new to the learner since it has been handled in area of plane figures in Form One. The emphasis here will be on the use of trigonometry. The learner should be involved in revision of trigonometric ratios.

Objectives

By the end of the topic, the learner should be able to:

(i) find the area of a quadrilateral.

(ii) find the area of other polygons (regular and irregular).

Time: Four lessons.

Teaching/Learning Activities

Area of Quadrilateral

- The learner should be involved in revision on area of simple plane figures.
- The teacher should involve the learner in revising area of triangle, as in example 2 in the students' book.
- The learner should be involved in solving problems such as the ones in examples 3, 4, 5 and 6.

Area of other Polygons (Regular and Irregular)

- The teacher should involve the learner in revision on polygons, sum of interior angles in relation to the number of sides, i.e., $(2n-4)90^0$.
- The teacher should help the learner to find the area of regular polygons, as in example 7.
- The learner should be given exercise 11.1.

Additional Hints

(i) For the area of parallelogram, consider a case where the two diagonals and an included angle are given.

(ii) The learner should be guided to establish that the area of a rhombus is given by; $A = \frac{1}{2}$ x product of its diagonals.

(iii) The learner should be made to realise that irregular polygons could be divided into one or more of the following shapes: rectangles, triangles, parallelograms and trapezia. The total area of the sub-divisions is then equivalent to area of the polygon.

(iv) The teacher should emphasise the use of logarithms of trigonometric ratios. Involve students in preparing cut-outs of various polygons.

(v) The following formula could also be used to find area of any regular polygon of n sides, each of length *l*;

$$A = \frac{1}{4}nl^2 \frac{1}{\tan\left(\frac{180}{n}\right)}$$

Evaluation

- Give a written test on area of quadrilaterals and other polygons. Discuss the test with the learners after marking.
- Present, on the chalkboard, random questions related to the topic. Have them attempted by the learners, then discussed in full.

Answers

Exercise 11.1

1. (a) 18.39 cm^2 (b) 34.77 cm^2 (c) 19 cm^2 (d) 11.4 cm^2
2. (a) 32.89 cm^2 (b) 36.05 cm^2 (c) 28.75 cm^2 (d) 13.26 cm^2
3. 59.86 cm^2
4. 28.8 cm^2
5. 10.66 cm^2
6. 6.8 cm
7. 43.4^0
8. 194.9 cm^2
9. 6 cm
10. 9.2 cm and 136.6^0

Chapter Twelve

AREA OF PART OF A CIRCLE

The learner has been taught fractions, area of a circle and area of a triangle. The teacher should involve the learner in revision of these topics. The teacher should use illustrations to define the terms 'sector' and 'segment'.

Objectives

By the end of the topic, the learner should be able to:

(i) find area of a sector.

(ii) find area of a segment.

(iii) find area of common region between two circles.

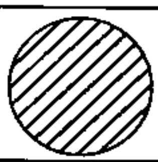
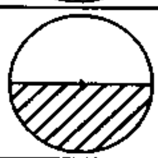
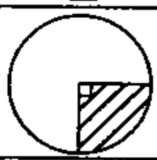
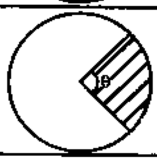
Time: Nine lessons.

Teaching/Learning Activities

Area of a Sector

The learner should be led into discovering the relationship between the angle subtended at the centre and the area of the sector containing the angle. The table below is useful.

Table 12.1

Figure	*Area*	*Angle at Centre*
	πr^2	360^0
	$\frac{180}{360}\pi r^2$	180^0
	$\frac{90}{360}\pi r^2$	90^0
	$\frac{\theta}{360}\pi r^2$	θ^0

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- The relationship established from the table can be summarised as; If θ is the angle subtended at the centre, then the area of the sector containing θ is given by $\frac{\theta}{360}\pi r^2$.
- The teacher should discuss examples 1, 2 and 3 in the students' book.
- The learner should be given exercise 12.1.

Area of a Segment

- The learner should be guided on how to find the area of a segment, as in example 4.
- The learner should be given exercise 12.2.

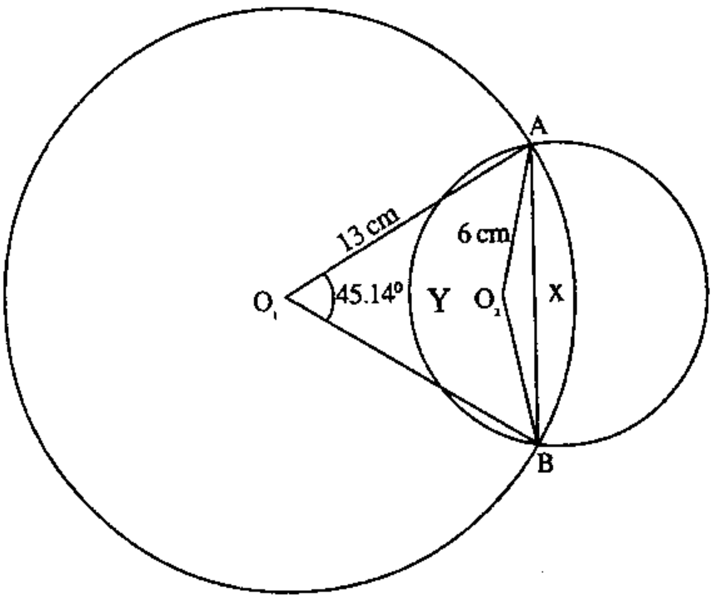
Area of Common Region Between Two Circles

- The learner should be led to find the area of common region between two circles, as in example 5. The teacher is advised to use teaching aids like relevant charts and circular cut-outs.
- The learner should be given exercise 12.3.

Additional Hints

The teacher should give more questions on common regions between two circles as the one below:

Find the area of the common region between two circles below, if $O_1A = 13$ cm, $O_2A = 6$ cm, $AB = 10$ cm, $\angle AO_1B = 45.14^0$ and $\angle AO_2B = 112.88^0$.



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Area	=	area of X + area of Y
Area of X	=	$\frac{45.14}{360}$ x 3.142 x 13^2 – $\frac{1}{2}$ x 13^2 sin 45.14
	=	66.58 – 59.90
	=	6.68
Area of Y	=	$\frac{112.88}{360}$ x 6^2 x 3.142 – $\frac{1}{2}$ x 6^2 sin 112.88
	=	35.46 – 16.58
	=	18.88

$\therefore$ Common area = 6.68 + 18.88

= 25.56 cm^2

Evaluation

- Give a written test on area of part of circle. Go through the test with the learners after marking.
- Present on the chalkboard related questions on the topic for further practice and discussion to ensure thorough understanding.

Answers

Exercise 12.1

1.	(a) 0.5131	(b) 1.7318	(c) 18.43	(d) 36.95
	(e) 17.34	(f) 36.16	(g) 262.7	
2.	1047.6	3. 150.8	4. 27.71	5. 346.4

Exercise 12.2

1.	(a) 8	(b) 64.35	(c) 48	(d) 16.35	(e) 97.85
2.	103.8				
4.	(a) 5	(b) 16.09	(c) 4.09		
5.	3.84				
6.	53.45				

Exercise 12.3

1.	(a) 15.59	(b) 49.75
2.	(a) 3.857	(b) 3.141
3.	71.46	
4.	80.07	

Chapter Thirteen

SURFACE AREA OF SOLIDS

The learner has already been introduced to common solids and area of plane figures, which will be useful in this topic. Use of nets in finding surface area of solids will be necessary.

Objectives

By the end of the topic, the learner should be able to:

(i) find the surface area of a prism.

(ii) find the surface area of a pyramid.

(iii) find the surface area of a cone.

(iv) find the surface area of a frustum.

(v) find the surface area of a sphere.

Time: Ten lessons.

Teaching/Learning Activities

Surface Area of a Prism

- With the knowledge already gained in form one on surface area of common solids, the teacher should guide the learner using a table similar to the one shown below, giving prisms of different cross-sections other than cubes and cuboids.

Table 1

Prism	*Shape of the base*	*Number of faces*
Triangular prism	Equilateral, isosceles or scalene	5
Hexagonal prism	Regular hexagon	8
Octagonal prism	Regular octagon	10
Heptagonal prism	Regular heptagon	9
Decagonal prism	Regular decagon	12

- The learner should be made to appreciate that the above table is not exhaustive.

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- The learner should now be involved in finding the surface area of these solids, as explained in the students' book, using the given dimensions.

Surface Area of Pyramid and a Cone

- The teacher should discuss how to find the surface area of a pyramid using examples 1 and 2 in the students' book. The distinction between the perpendicular and slanting height should be brought out clearly.
- The teacher should involve the learner in the project work highlighted in the students' book. The learner should then be guided on how to derive the formula for finding the surface area of a cone.
- The teacher should show the applications of the formula, as in example 3.

Surface Area of a Frustum

- The teacher should define the term 'frustum'. The learner should be guided in the project work in the students' book.
- The learner should then be guided on how to find the surface area of a frustum using example 4 in the students' book.

Surface Area of a Sphere

- The learner is meeting the term 'sphere' for the first time. The teacher should give examples of solids with spherical shapes like the globe, a ball and shot putt.
- The teacher should then state the formula for finding the surface area of a sphere and discuss its application as in example 5.
- The learner should then be given exercise 13.1.

Additional Hints

(i) The teacher should also discuss how to find the surface area of a tetrahedron with different triangular base.

(ii) The teacher should discuss the surface area of a hemisphere. Examples in real life situations should be given, for example, a kitchen bowl and a mortar.

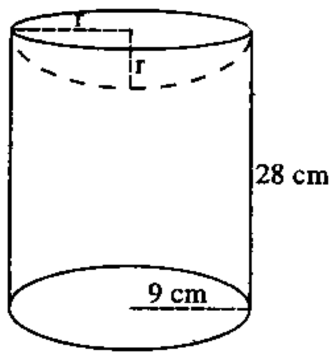
Evaluation

- Give a written test on surface area of solids. Discuss the paper with the learners after marking to ensure all concepts are understood.

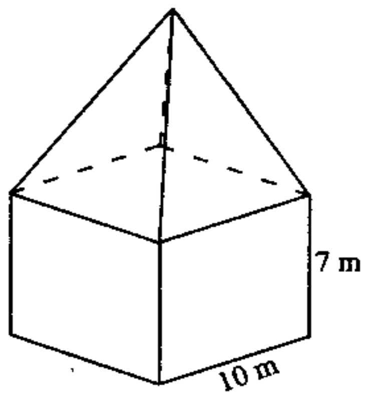
- For further practice and discussion, present random questions on the topic on the chalkboard and have the learners attempt them.

Further Questions

1. A pyramid 8 cm high stands on a base which is a regular hexagon ABCDEF of side 5 cm. Find the surface area of the pyramid.
2. Find the surface area of a tetrahedron on an equilateral base of sides 8 cm whose slanting edges are 11 cm long.
3. A cylindrical solid which is open at the top has a radius of 9 cm and a height of 28 cm. If a portion of the cylinder is cut off along the dotted line as shown below, find the surface area of the remaining part.



4. The barn shown below is 10 m square and has walls 7 m high. The roof rises to a point 10 m above the ground. Find its surface area.



5. A dog kennel is 90 cm long and 75 cm wide as shown in the figure below. The walls are 60 cm high and the roof is hemispherical in shape, the highest point being 75 cm from the ground. Find the total are of the kennel in m^2.

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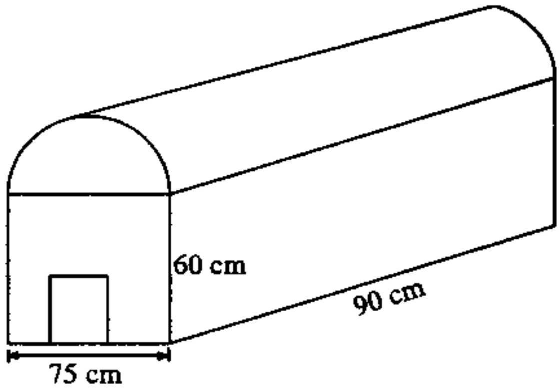


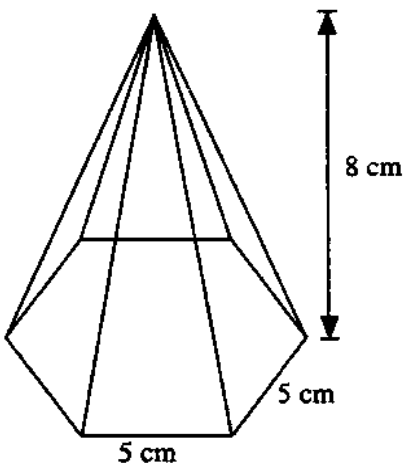
Answers

Exercise 13.1

1. (a) $534.14\ cm^2$ (b) $528\ cm^2$ (c) $207.81\ cm^2$ (d) $56.56\ cm^2$
2. $1.8568\ m^2$
3. $113.1\ m^2$
4. $105.88^0 \bumpeq 105.9^0 \pm 0.1^0$
5. $88.78\ cm^2$
6. $706.95\ cm^2 \bumpeq 70.7\ cm^2$
7. (a) $312.4\ m^2$ (b) $245.99 m^2 \bumpeq 246\ m^2$
8. $10.49\ m^2$
9. $13751.4\ cm^2$
10. $3\ 850\ m^2$
11. $191.0\ cm^2$
12. $1080.5\ cm^2$

Further questions

1.



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Base area = area of one equilateral triangle x 6

$= (\frac{1}{2} \text{ ab sin } 60^0)\ 6$

$= \frac{1}{2} \times 5 \times 5 \times 0.8660 \times 6$

$= 64.95 \text{ cm}^2$

Slanting edge $= \sqrt{(8^2 + 5^2)}$

$= 9.434 \text{ cm}$

Slanting height of each triangular face $= \sqrt{(9.434^2 - 2.5^2)}$

$= 9.097 \text{ cm}$

Area of the six triangle faces = area of one triangle x 6.

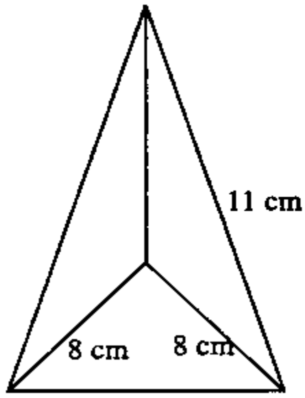
$= (\frac{1}{2} \times 5 \times 9.097) \times 6$

$= 136.5 \text{ cm}^2$

Therefore surface area $= (64.95 + 136.5) \text{ cm}^2$

$= 201.5 \text{ cm}^2$

2.



Surface area = base area + area of 3 triangular faces.

Base area $= (\frac{1}{2} \text{ ab sin } 60^0)$

$= \frac{1}{2} \times 8 \times 8 \times \sin 60^0$

$= \frac{1}{2} \times 8 \times 8 \times 0.8660$

$= 27.72 \text{ cm}^2$

Height of one triangle $= \sqrt{(11^2 - 4^2)}$

$= \sqrt{(121 - 16)}$

$= 10.25 \text{ cm}$

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Area of three slanting faces $= \text{area of one triangle} \times 3.$

$= \frac{1}{2} \times 8 \times 10.25 \text{ cm}^2$

$= 41 \text{ cm}^2$

$\therefore$ Surface area $= (27.72 + 40.99) \text{ cm}^2$

$= 68.71 \text{ cm}^2$

3. Surface area of the remaining part is:

Surface area of remaining part = area of whole cylinder − area of hemispherical part

$= (\pi r^2 + 2\pi rh) - \frac{1}{2} \times 4\pi r^2$

$= [(3.142 \times 9 \times 9) + (2 \times 3.142 \times 9 \times 28) - [(2 \times 3.142 \times 9 \times 9)]$

$= [(254.50 + 1583.57)] - 509.0$

$= 1838.07 \text{ cm}^2 - 509.0 \text{ cm}^2$

$= 1\,329 \text{ cm}^2$

4. Surface area is area of the base plus area of the walls plus area of the top.

Surface area $= (10 \times 10) + (10 \times 7)\, 4 \text{ cm}^2$

$= 100 + 70 \times 4$

$= (100 + 280) \text{ cm}^2$

$= 380 \text{ cm}^2$

Length of the diagonal $= \sqrt{10^2 + 10^2}$

$= \sqrt{200}$

$= 14.14 \text{ cm}$

Length of slanting edge at the top $= \sqrt{(7.07^2 + 3^2)}$

$= \sqrt{58.98}$

$= 7.68 \text{ cm}$

Slanting height $= \sqrt{7.68^2 - 5^2}$

$= \sqrt{33.98}$

$= 5.83 \text{ cm}$

Area of the top = area of one triangle x 4

Area $= (\frac{1}{2} \times 10 \times 5.83)\, 4 \text{ cm}^2$

$= 29.15 \times 4$

$= 116.59 \text{ cm}^2$

Therefore, surface area $= 380 \text{ cm}^2 + 116.59 \text{ cm}^2$

$= 496.59 \text{ cm}^2$

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Chapter Fourteen

VOLUME OF SOLIDS

The learner has met calculation of volumes of regular solids such as cuboids and cylinders. This knowledge will be extended to finding volumes of prisms, pyramids, cones, frustum and sphere.

Objectives

By the end of the topic, the learner should be able to:

(i) find the volume of a prism.

(ii) find the volume of a pyramid.

(iii) find the volume of a cone.

(iv) find the volume of frustum.

(v) find the volume of a sphere.

Time: Twelve lessons.

Teaching/Learning Activities

Volumes of Prisms, Pyramid and Cone

- The teacher should involve the learner in the revision on volumes of cubes, cuboids and cylinders.
- The learner should be guided in finding the volume of a prism by discussing examples 1 and 2.
- The learner should be given exercise 14.1.
- The learner should be guided in carrying out the project work and led through examples 3, 4 and 5.
- The learner should do exercise 14.2 numbers 1 – 10 and 12.

Volume of a Frustum

- Similarity, linear scale factor and volume scale factor should be revised. This will enable the students to get the volume of the larger cone and that of the smaller cone that is cut off to make the frustum.
- The learner should be guided through example 6.
- The learner should be given questions 11, 13 and 14 of exercise 14.2.

Volume of a Sphere

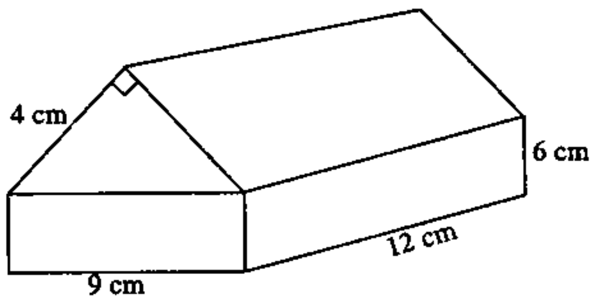
- The teacher should introduce the use of formula $V = \frac{4}{3}\pi r^3$ for finding the volume of sphere, as in example 7.
- The learner should be given exercise 14.3.

Evaluation

- Give a written test on volumes of solids. Discuss the paper with the learners after marking.
- Present related questions to the learners for further practice. After they have attempted them, have the same discussed, so that all concepts are clearly understood.
- Have the learners do mixed exercises to assess them on topics met so far. Ensure the exercise is discussed in detail after marking.

Further Questions

1. A container is made from a metal pipe $\frac{1}{2}$ cm thick and of external diameter 8 cm, by welding a flat disc of the same metal $\frac{1}{2}$ cm thick and 8 cm diameter to one end. What is the volume of the metal used to make the container if the pipe is 6 cm long?
2. A hemispherical container has a radius of 12 cm. Find its capacity in litres.
3. A trench 50 m long has a trapezoidal cross-section 1.8 m wide at the top, 1.2 m wide at the bottom and 1.5 m deep. What volume of earth was excavated during its construction?
4. The figure below shows a model of a building. Find the amount of space occupied by the model.



5. A cylindrical container is 1.05 m in diameter and contains water to a depth of 1.2 m. If a stone is immersed into the water, the level rises to a height of 1.7 m. Find the volume of the stone.

Answers

Exercise 14.1

1. 27 m^3
2. 11 880 cm^3
3. (a) 24.57 m^2 (b) 61.42 m^3 (c) 61 420 litres
4. (a) 10.392 cm^2 (b) 20 784 cm^3 (c) 162.1 kg
5. 432 m^3
6. (a) 9 256 m^2 (b) 185122.8 cm^3
7. 41.04 m^3
8. (a) 1 000 m^3 (b) 1 000 000 litres

Exercise 14.2

1. (a) 240 cm^3 (b) 120 cm^3 (c) 320 cm^3
2. (a) 5 184 cm^3 (b) 120.68 cm^3 (c) 149.88 cm^3
3. (a) 10 cm, 161.3 cm^2 (b) 12 cm
4. 15 cm; 15.4 cm; 177 cm^2
5. 6 cm
6. 896 cm^3
7. 8 662.1 cm^3
8. 2 688 cm^3
9. 1.23 cm
10. (a) 36 cm^3 (b) 432 π cm^3 (c) 67.9 cm^3
    (d) 560π cm^3 (e) 648 π cm^3
11. 426.78 cm^2, $485\frac{1}{3}$ cm^3
12. (i) 126.4 cm^2 (ii) 111.26 cm^3
13. $\frac{h^3 - (h - t)^3}{h^3}$
14. (a) 470.7 cm^3 (b) 216 cm^3 (c) 61.13 cm^3

Exercise 14.3

1. (a) 463.43 cm^3 (b) 38 808 cm^3
2. 56 spheres
3. 908 cm^3
4. 46.3 cm
5. 1 cm
6. 161.2 kg

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7. 128.6 kg
8. 93.75 cm
9. 48 600 m (question should read 'diameter 36 cm')

Further Questions

1. Volume $= \text{volume of pipe} + \text{volume of disc}$

$= (\pi r^2 h - \pi r^2 h) + (\pi r^2 \times \frac{1}{2})$

$= ([3.142 \times 4 \times 4 \times 6) - (3.142 \times 3.5 \times 3.5 \times 6)] + 3.142 \times 4 \times 4 \times \frac{1}{2}$

$= (301.63 - 230.94) + 25.14$

$= 95.83 \text{ cm}^3$

2. Volume $= \frac{1}{2}(\frac{4}{3}\pi r^3) \text{ cm}^3$

$= \frac{1}{2}\left(\frac{4 \text{ X } 3.142 \text{ X } 12 \text{ X } 12 \text{ X } 12}{3}\right)$

$= 3619.58 \text{ cm}^3$

Therefore, capacity $= \frac{3619.58}{1000}$

$= 3.62$ litres

3. Volume $= \text{area of cross-section x height}$

$= \frac{1}{2}(1.8 + 1.2)1.5 \text{ x } 50 \text{ cm}^3$

$= \frac{1}{2}(3 \text{ x } 1.5) \text{ x } 50$

$= 2.25 \text{ x } 50$

$= 112.5 \text{ m}^3$

4. Volume $= \text{volume of the top} + \text{volume of the bottom}$

$= [9 \text{ x } 12 \text{ x } 6] + (\frac{1}{2} \text{x } 4 \text{ x } 8.062)12 \text{ cm}^3$

$= 648 + 193.5$

$= 841.5 \text{ cm}^3$

5. Volume of the stone $= \pi r^2 \text{ x } h$

$= 3.142 \text{ x } \frac{1.05}{2} \text{ x } \frac{1.05}{2} \text{ x } 0.5$

$= 0.433 \text{ m}^3$

Mixed Exercises 2

1. 234
2. (a) 36^0 (b) 1.386 cm^2
3. 368 cm^2
4. $a = 2, b = 1$
5. 5450.9 cm^3

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6. (a) 226.19 cm^2 (b) 82.19 cm^2
7. 285 cm^2
8. 7.684 km
9. (a) 8.485 cm (b) 8.485 cm (c) 10.39
10. (a) 6.5 cm (b) 18.2 cm
11. (a) 386.2 cm^2 (b) 332.56 cm^3
12. 66.94^3
13. (a) 20^0 (b) 0.3640
14. 79.19
15. 1953.7 cm^3
16. 880 cm^2, r = 7 cm, h = 13 cm
17. (a) CJ = 5.66 cm, DC = 8 cm (b) AGH : ABH = 1 : 2
    (c) 1 : 2 (d) V= 271.5 cm^2, S.A. = 293.02 cm^2
18. $2\frac{1}{2}$ cm
19. 84.86 cm^2
20. 1.27 m
21. 128.3 cm^3
22. 5.878 cm
23. (a) 642.8 m (b) 766.0 m
24. AB = 96 cm, AC = 83 cm
25. 73.34 m
26. 336.9 cm^3
27. 31.51 m
28. 139.5 m, 1.995 km
29. 95.41 cm^3, 144.51 cm^2
30. 7.52 m
31. (a) x = 14 cm, y = 10.72 cm (b) x = y = 6.36 cm
    (c) x = 5.9 cm, y = 3.12 cm
32. 23 cm, 79.67 cm
33. 8.41 cm, 11.56 cm
34. 3.71 cm
35. 83.65 cm^2
36. 34 *l*
37. (a) (i) 6.47 cm (ii) 4.70 cm (iii) 5.19 cm (iv) 47.8^0
38. 6.2 cm
39. 43.67 cm 40. 136636.4 cm^2

Chapter Fifteen

QUADRATIC EXPRESSIONS AND EQUATIONS

The learner has met simple cases of expanding algebraic expressions such as $2(a + b)$, $a(a - b)$, etc. However, it will be helpful to revise these cases before discussing expansions that lead to quadratic expressions.

It is also useful to revise factorisation of simple algebraic expressions, such as $2x + 2y$, $3a - 12b$, etc.

Objectives

By the end of the topic, the learner should be able to:

(i) expand algebraic expressions that form quadratic equations.
(ii) derive the three quadratic identities.
(iii) identify and use the three quadratic identities.
(iv) factorise quadratic expressions including the identities.
(v) solve quadratic equations by factorisation.
(vi) form and solve quadratic equations.

Time: Twelve lessons.

Teaching/Learning Activities

Expansion of Algebraic Expressions

- The teacher should guide the learner in expanding algebraic expressions, as in the students' book.
- The teacher should lead the learner through examples 1 and 2.
- The learner should be given exercise 15.1.

Derivation of the Three Identities

- The learner should be guided to derive the three identities using the idea of area. The following illustrations are useful to the teacher in deriving the identities;

  (i) $(a + b)^2$

  The area of the square whose side is $(a + b)$ cm is; $(a + b)(a + b)$ cm^2, see figure 15.1.

  The area of the square is also equal to the area of A plus area of B plus area of C plus area of D.

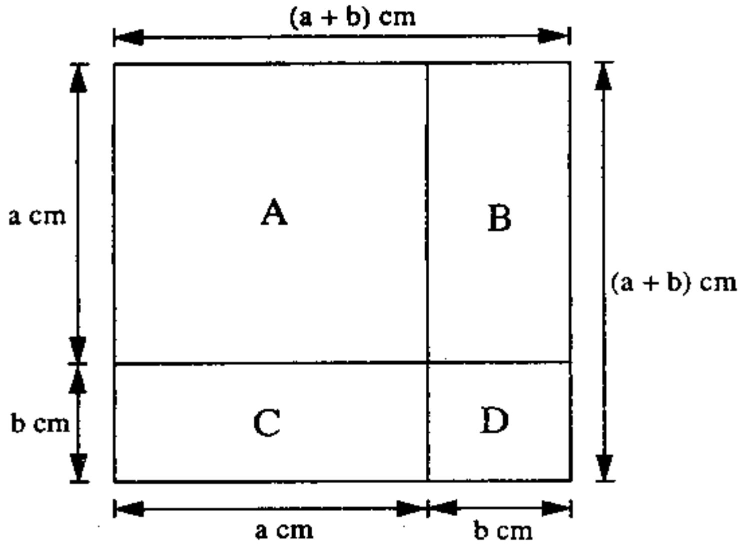

Fig. 15.1

$$\text{Thus, area of the square} = (a^2 + ab + ab + b^2)\ cm^2$$
$$= (a^2 + 2ab + b^2)\ cm^2$$

$(a + b)(a + b) = a^2 + 2ab + b^2$

Therefore, $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2$

Consider figure 15.2.

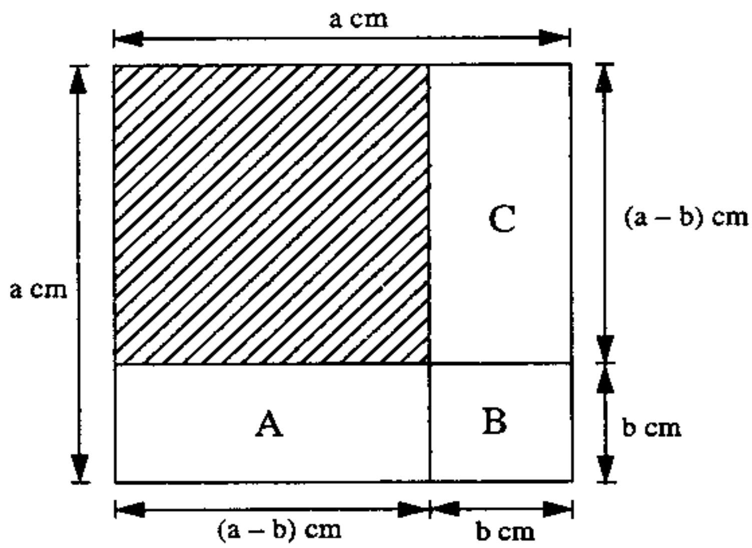

Fig. 15.2

Area of the shaded region $= (a - b)(a - b)\ cm^2$

Area of the square whose side is a cm $= a^2\ cm^2$

Area of the unshaded region $= \text{area of A} + \text{area of B} + \text{area of C}$
$= b(a - b) + b^2 + b(a - b)$
$= ab - b^2 + b^2 + ab - b^2$
$= 2ab - b^2 \text{ cm}^2$

Area of the shaded region $= a^2 - (2ab - b^2) \text{ cm}^2$
$= (a^2 - 2ab + b^2) \text{ cm}^2$

Therefore, $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)(a - b)$

Consider figure 15.3.

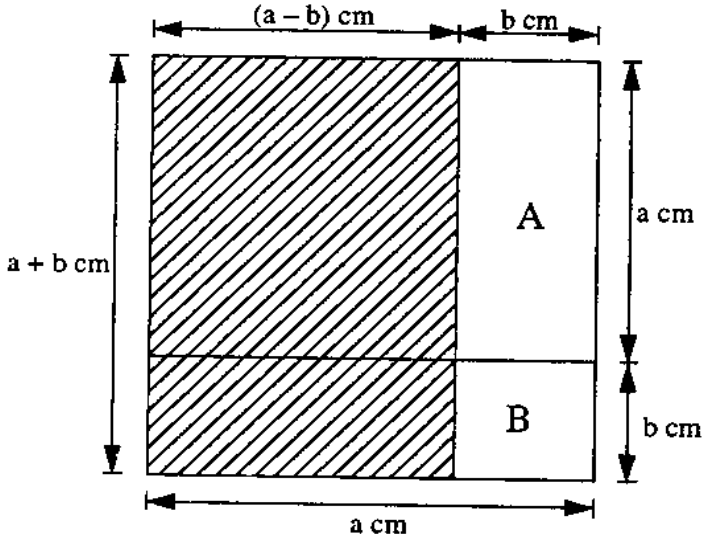

Fig. 15.3

Area of the shaded region is given by $(a + b)(a - b) \text{ cm}^2$.
Also, area of the shaded region is equal to the area of the rectangle whose length is $(a + b)$ cm and width a cm minus the area of the unshaded region.

Area of the rectangle $= a(a + b) \text{ cm}^2$

Area of unshaded region $= \text{area of A} + \text{area of B}$
$= (ab + b^2) \text{ cm}^2$

The area of shaded region $= a(a + b) - (ab + b^2)$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

Therefore, $(a + b)\ a - b) = a^2 - b^2$

Identification and Use of Quadratic Identities

- The teacher should guide the learner to establish the identities using expansion, as in the students' book.

- The learner should be led to solve numerical problems using quadratic identities, as in example 3.
- The learner should be given exercise 15.2.

Factorisation of Quadratic Expressions

- The learner should be guided to factorise quadratic expressions when the coefficient of x^2 is 1, as in the students' book.
- The teacher should discuss example 4.
- The learner should be given exercise 15.3.
- The learner should be guided to factorise quadratic expressions when the coefficient of x^2 is greater than one.
- The teacher should guide the learner through example 5.
- The learner should be given exercise 15.4.

Solution to Quadratic Equations by Factorisation

- The teacher should guide the learner through examples 6 and 7.
- The learner should be given exercise 15.5.

Formation and Solution of Quadratic Equations

- The learner should be led to form quadratic equations from given roots, as in the students' book.
- The learner should be given exercise 15.6.
- The teacher should guide the learner through example 7.
- The learner should be given exercise 15.7.

Additional Hints

(i) While introducing expansion of algebraic expressions, the following may be useful;

- Encourage the learner to expand by one term at a time, e.g;

$$\begin{aligned}(7x + 5)(4x - 3) &= 7x(4x - 3) + 5(4x - 3)\\ &= 28x^2 - 21x + 20x - 15\\ &= 28x^2 - x - 15\end{aligned}$$

- Using the long multiplication method;

$(7x + 5)(4x - 3)$

$$\begin{array}{rr} = & 7x + 5 \\ \times & \underline{4x - 3} \\ & 28x^2 + 20x \\ + & -21x - 15 \\ \hline & \underline{28x^2 - x - 15} \end{array}$$

(ii) Solving of quadratic equations should be limited to cases where factors can be found.

(iii) Note the following common errors:

- Failure to distinguish between quadratic expressions and quadratic equations.
- Wrong cancellation when simplifying algebraic fractions.

Evaluation

- Give a written test on quadratic expressions and equations. Discuss the test with the learners after marking.
- Give related questions from other sources for further practice and discussion. Have the learners attempt them to enhance their understanding of the topic.

Further Questions

1. Work out 1 012 x 988 by regarding it as (1 000 + 12)(1 000 – 12)
2. Calculate 96^2 by factorising $96^2 - 4^2$
3. Simplify $\frac{2x^2 + 3x + 1}{x^2 - 1}$

Answers

Exercise 15.1

1. (a) $24x^2 + 26x + 6$ (b) $3x^2 + 13x - 10$
   (c) $x^2 + x - 2$ (d) $2x^2 + x - 6$
   (e) $12x^2 + 44x - 16$ (f) $-x^2 - 2x + 8$
2. (a) $x^2 + 15x + 56$ (b) $6x^2 + 5x - 6$
   (c) $15x^2 + 34x + 15$ (d) $28x^2 - 65x + 28$
   (e) $-m^2 + 2mn - n^2$ (f) $2x^2 + 3x - 9$
3. (a) $2a^2x^2 - abx - 3b^2$ (b) $2x^2 - 5x + 2$
   (c) $30x^2 + 11x - 30$ (d) $y^2 - 4$
   (e) $x^2 + 4a + 4a^2$ (f) $15x^2 - 38x + 24$
4. (a) $x^2 - 4a^2$ (b) $x^2 - 49$
   (c) $x^2 + 4x + 4$ (d) $16x^2 + 16x + 4$
5. (a) $\frac{1}{4} + x + x^2$ (b) $a^2x^2 + 2adx + d^2$
   (c) $\frac{1}{16} + \frac{1}{2x} + \frac{1}{x^2}$ (d) $x^2 - 6x + 9$
   (e) $16x^2 - 6x + \frac{9}{16}$ (f) $1 - \frac{-2}{x} + \frac{1}{x^2}$

Exercise 15.2

1. (a) $x^2 + 10x + 25$ (b) $x^2 - 10x + 25$
   (c) $16x^2 + 24x + 9$ (d) $4x^2 + 12x + 9$
2. (a) $9 - 6x + x^2$ (b) $\frac{1}{4} + x + x^2$
   (c) $\frac{1}{x^2} - \frac{2}{x} + 1$ (d) $\frac{1}{16} + \frac{3}{8}b + \frac{9}{16}b^2$
3. (a) $\frac{1}{4}a^2 - \frac{1}{3}ab + \frac{1}{9}b^2$ (b) $1 - \frac{1}{a^2}$
   (c) $\frac{1}{x^2} - \frac{1}{y^2}$ (d) $4x^2 - 9y^2$
4. (a) $\frac{1}{9y^2} + \frac{1}{6yx} + \frac{1}{16x^2}$ (b) $\frac{1}{9}x^2 + \frac{2}{3}x + 1$
   (c) $\frac{1}{25}a^2 + \frac{8}{25}ab + \frac{16}{25}b^2$ (d) $\frac{9}{49}x^2 - \frac{3}{7}xy + \frac{1}{4}y^2$

Exercise 15.3

1. (a) $(x + 2)(x + 4)$ (b) $(x - 2)(x - 3)$
   (c) $(x - 3)(x + 7)$ (d) $(x + 2)(x - 1)$
   (e) $(x - 5)(x + 7)$ (f) $(x - 6)(x - 4)$
2. (a) $(x - 8)(x + 4)$ (b) $(x + 9)(x - 6)$
   (c) $(x + 1)^2$ (d) $(x + 2)^2$
   (e) $(x - 1)^2$ (f) $(x - 7)^2$
3. (a) $(x - 8)^2$ (b) $(x + 2)(x - 2)$
   (c) $(1 + x)(1 - x)$ (d) $2(x + 4)(x - 4)$
   (e) $3(x - 2)(x - 3)$ (f) $(x - 6)(x + 1)$
4. (a) $(x + 2)(x + a)$ (b) $t(t + 2)(t + 6)$

Exercise 15.4

1. (a) $(x + 2)(2x - 1)$ (b) $(x - 2)(3x + 4)$
   (c) $(x + 1)(4x + 3)$ (d) $(x - 4)(5x - 1)$
2. (a) $2(x - 1)(7x - 1)$ (b) $(x + 3)(3x + 2)$
   (c) $(x + 2)(4 - 3x)$ (d) $2(x + 2)(x + 6)$
3. (a) $(7x + 3)(5x + 4)$ (b) $(7x - 3)(7 - 3x)$
   (c) $(15x - 7)(10x + 3)$ (d) $(17x - 3)(x + 5)$
4. (a) $(2x + 1)(4x + 1)$ (b) $(x + 9)(x - 13)$
   (c) $(3x + 8)^2$ (d) $(5x + 3)(15x - 7)$
5. (a) $(10x + 1)(1 - 3x)$ (b) $(x - 1)(19x - 3)$
   (c) $(7x + 9)(9x + 7)$ (d) $(x - \frac{1}{3})(x + \frac{1}{5})$
6. (a) $(2x - 3)^2$ (b) $(3x + 1)^2$

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(c) $2(x+4y)(x-4y)$ (d) $(\frac{1}{y}-1)^2$

7. (a) $t^4(t^2+1)(t-1)(t+1)$ (b) $(7x+4)(5x-3)$
   (c) $(2x+\frac{1}{2})^2$ (d) No integral factors
8. (a) $(x+1)(21x-4)$ (b) $(5x-\frac{1}{4})^2$
   (c) $(1-x)(29x+3)$ (d) $(\frac{1}{4}x-7)^2$
9. (a) $(3x-8)(8x+7)$ (b) $(3x-2)(11x+2)$
   (c) $(\frac{1}{x}-2)^2$ (d) $(t^2+w^2)[(w+t)(w-t)+1]$
10. (a) $(5x+y)^2$ (b) $(x+y)(x+y+1)+1$
    (c) $(2x+3)^2$ (d) No integral factors

Exercise 15.5

1. (a) $x=-2$ or -4 (b) $x=\frac{1}{2}$ or -2
   (c) $x=1$ or $\frac{1}{7}$ (d) $x=-1$ or $-\frac{4}{3}$
2. (a) $x=5$ or -5 (b) $x=-1$ or $\frac{5}{3}$
   (c) $x=0$ or 1 or $-\frac{4}{3}$ (d) $x=\frac{1}{3}$ or $-\frac{1}{2}$
3. (a) $x-\frac{1}{2}$ or $-\frac{1}{4}$ (b) $x=5$ or 4
   (c) $x=-2$ or -6 (d) $\frac{3}{2}$ or $-\frac{3}{2}$
4. (a) $x=\frac{1}{4}$ or $-\frac{3}{4}$ (b) $x=\frac{3}{2}$ or $-\frac{1}{2}$
   (c) $x=\frac{1}{6}$ or -1 (d) $x=\frac{1}{7}$ or $-\frac{1}{5}$
5. (a) $x=3$ (b) $x=\sqrt{\frac{11}{5}}$ or $-\sqrt{\frac{11}{5}}$
   (c) $x=6$ or -2 (d) $x=1$ and $y=1$
6. (a) $x=1$ or $\frac{8}{7}$ (b) $x-\frac{1}{2}$ or $-\frac{1}{4}$
   (c) $x=\frac{1}{3}$ or $-\frac{1}{5}$ (d) $x=5$ or $-\frac{3}{2}$
7. (a) $x=4$ or $\frac{1}{2}$ (b) $x=-1$
   (c) $x=\frac{1}{3}$ or $-\frac{1}{6}$ (d) $y=2$
8. (a) $x=8$ or -2 (b) $x=-1$ or $-\frac{1}{3}$

Exercise 15.6

1. $x^2-4=0$
2. $6x^2-5x+1=0$

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3. $4x^2 + 11x - 3 = 0$
4. $x^2 + 8x + 15 = 0$
5. $8x^2 - 10x + 3 = 0$
6. $x^2 + 3x = 0$
7. $x^2 + 6x + 9 = 0$
8. $4x^2 - 4x - 35 = 0$
9. $x^2 - x(p + q) + pq = 0$
10. $pqx^2 - x(p + q) + 1 = 0$

Exercise 15.7

1. –1 or 4
2. 3 cm
3. 5 cm
4. Length 6 cm, width 3cm
5. 20 people
6. Area of larger circle is 100 cm^2
7. x = 1 cm
8. Sh. 8 280
9. 14 m^2
10. 40 students
11. 25 members
12. 156^0, 12^0, 12^0 and 9^0, 9^0, 162

Further Questions

1. $1\,012 = 1\,000 + 12$

   $988 = 1\,000 - 12$

   Therefore, $1\,012 \times 988 = (1\,000 + 12)(1\,000 - 12)$

   $$= 1\,000^2 - 12^2$$
   $$= 1\,000\,000 - 144$$
   $$= 999\,856$$

2. $96^2 - 4^2 = (96 + 4)(96 - 4)$

   $$= 100 \times 92$$
   $$= 9\,200$$
   $$= 9\,200$$

   Therefore, $96^2 = 9\,200 + 4^2$

   $$= 9\,200 + 16$$
   $$= 9\,216$$

$$2x^2 + 3x + 1 = 2x^2 + 2x + x + 1$$
$$= 2x(x + 1) + 1(x + 1)$$
$$= (2x + 1)(x + 1)$$
$$x^2 - 1 = (x + 1)(x - 1)$$

Therefore, $\dfrac{2x + 3x + 1}{x^2 - 1} = \dfrac{(2x + 1)(x + 1)}{(x + 1)(x - 1)}$
$$= \frac{2x + 1}{x - 1}$$

Chapter Sixteen

LINEAR INEQUALITIES

The learner has met the symbols ‘>’ meaning ‘greater than’ and also ‘<’ meaning ‘less than’ in primary school. He/she has learnt the ordering of integers and knows that a number to the left (on a number line) is less than a number to the right of it. The learner is also familiar with the symbol ‘=’ meaning ‘equal’ to. The teacher should revise <, > and =, then combine them to get ≥ or ≤.

Objectives

By the end of topic the learner should be able to:

(i) identify and use inequality symbols.
(ii) illustrate inequalities on the number line.
(iii) solve linear inequalities in one unknown.
(iv) solve inequalities in two unknowns graphically.
(v) form simple linear inequalities from inequality graphs

Time: Twelve lessons.

Teaching/Learning Activities

Inequality Symbols

- The teacher should revise with the learner the mathematical symbols >, < and =, as in the students’ book. Heuristically, the learner should be guided to define the term ‘inequality’.
- Examples from real life where inequalities apply should be discussed and given to the learner to form inequalities, e.g.,
  (i) The width of the chalkboard, w metres, is less than 1.5 m.
  (ii) In 6 years time, Kamau will be over 30 years of age. Write an inequality in x if Kamau’s age now is x years.
  (iii) The standard depth of pit latrines in Kenya, d metres must not be less than 10 metres.
  (iv) The area A square metres allocated to each sheep for grazing by a farmer is almost 50 square metres.

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Inequalities on the Number Line

- The convention on shading the region satisfied by an inequality should be discussed as in the students' book.
- Representation of compound statements on the number line should also be discussed as in the students' book.
- The learner should be given exercise 16.1.

Solution of Inequalities in One Unknown

- The learner should be led to establish that:
  - (i) adding or subtracting a constant on both sides of an inequality does not reverse the inequality symbol, as indicated in examples 2 and 3 in the students' book.
  - (ii) Multiplying or dividing by a positive number does not reverse the inequality symbol, as in examples 4 and 5.
  - (iii) Multiplying or dividing by a negative reverses the inequality symbol, as in example 6.
- The learner should be given exercise 16.2.
- The teacher should lead the learner in the discussion of solutions to simultaneous inequalities.
- The learner should be led through example 7 in the students' book.
- The learner should be given exercise 16.3.

Graphical Representation of Linear Inequalities

- The teacher should guide the learner through the shading conventions and the regions so formed be clearly defined with great emphasis on drawing of boundary lines of regions, as in figure 16.8.
- The teacher should go through figure 16.9, lead the learner in finding linear inequalities for a given graph.
- The learner should be given exercise 16.5.
- The learner should be led to find a region satisfied by a set of inequalities, as in examples 9 and 10.
- The learner should be given exercise 16.6.

Additional Hints

(i) When the boundary line is not passing through the origin, the learner should be encouraged to use the origin to test the point (O, O) during inspection to find the required region.

(ii) Emphasis should be laid on the shading of unwanted region.

(iii) The learner should be encouraged to use the y and x intercepts when graphing boundary lines.

(iv) The teacher should expose the learner to situations where the latter is required to state integral values in a range, e.g., stating the integral values which satisfy the inequality $-0.5 \leq x \leq 5.5$. The integral values here are; 0, 1, 2, 3, 4 and 5

Evaluation

- Give a written test on linear inequalities. Discuss the paper with the learners after marking.
- Get related questions from other sources and have the learners attempt them. Have the same discussed thereafter.

Answers

Exercise 16.1

1. Check for accuracy of drawings.
2. Check for accuracy of drawings.
3. Check for accuracy of drawings.

Exercise 16.2

1. (a) $2 < x < 5$ (b) $3 \geq x < 6$ (c) $1 \leq x \leq 7$
   (d) $-4 < x \leq 0$ (e) $-3 \leq x \leq -1$

2,3. Check for accuracy of drawings.

Exercise 16.3

1. (a) $x > 3$ (b) $x < \frac{7}{3}$
2. (a) $x > \frac{1}{5}$ (b) $x \leq -9$
3. (a) $x \geq 4$ (b) $x \leq -2$
4. (a) $x < -12$ (b) $x > -7$
5. (a) $x \geq \frac{13}{3}$ (b) $x \geq 15$
6. (a) $x < -2$ (b) $x \leq -2$
7. (a) $x \geq 6$ (b) $x \geq 343$

Check for accuracy of drawings.

Exercise 16.4

1.	(a) $2 < x < 8$	(b) $-4 \le x \le 5$
2.	(a) $-6 \le x < -1$	(b) $-1 < x \le 9$
3.	(a) $-1 < x < 1$	(b) $1\frac{1}{2} < x < 2$
4.	(a) $\frac{32}{35} < x < \frac{10}{7}$	(b) $1 \le x < 4$
5.	(a) $-2 \le x < 4$	(b) $x < -6$
6.	(a) $x < 6$	(b) $3 \le x < 15$
7.	(a) $\frac{2}{5} \le x \le 2$	(b) $3\frac{1}{2} \le x \le 7$
8.	(a) $-\frac{3}{2} < x$	(b) $-\frac{2}{3} \le x < \frac{3}{2}$
9.	(a) $3 < x \le 5$	(b) $4 < x < 2$

Check for accuracy of drawings.

Exercise 16.5

Check for accuracy of graphs.

Exercise 16.6

Check for accuracy of graphs.

18	(a)	$y \le -x + 2$	(b)	$y \ge \frac{4}{5}x + 4$
	(c)	$y > 2$	(d)	$x > -2$
	(e)	$y < 5x + 5$	(f)	$x < 3$

Exercise 16.7

Check for accuracy of graphs.

6. (a) 7.5 sq units (b) 12 sq. units
7. Any four points with integral co-ordinates in the unshaded region.
8. $y \le x$, $y \ge 0$, $x \le 8$
9. (a) D $(0, -5)$
   (b) $x + y = 5$; $y = x + 5$; $y + x = -5$
   $y - x = -5$
   (c) $x + y \le 5$; $y \le x + 5$; $y + x \ge -5$, $y \ge x - 5$
10. $2y \ge x$, $y \ge x - 3$, $y + 7x \le 61$; $7y - 12x \le 0$
11. $y \ge 2x - 2$; $y + x \ge -2$

Chapter Seventeen

LINEAR MOTION

This topic is not entirely new. The learner has met problems related to distance, speed and time at primary level and in form one. The knowledge of graphs is an important pre-requisite.

Objectives

By the end of the topic, the learner should be able to:

(i) define displacement, speed, velocity and acceleration.
(ii) distinguish between distance and displacement, speed and velocity.
(iii) determine velocity and acceleration.
(iv) plot and draw graphs of linear motion (distance-time and velocity- time graphs).
(v) interpret graphs of linear motion.
(vi) define relative speed.
(vii) solve problems involving relative speed.

Time: Ten lessons.

Teaching/Learning Activities

Definition of Terms

- The teacher should lead the learner through the definition of displacement, speed, velocity and acceleration, giving appropriate units as in the students' book.
- The learner should be led to define average speed as in the students' book.
- The learner should be given exercise 17.1.

Velocity and Acceleration

- The teacher should help the learners to define the terms average velocity and acceleration.
- The learner should be given exercise 17.2.

Distance – Time and Velocity – Time graphs

- The learner should be guided to draw distance-time and velocity- time graphs, as in the students' book.

- The teacher should discuss how to interpret the graphs using the illustrations in the students' book.
- The learner should be given exercises 17.3 and 17.4.

Relative Speed

- The teacher should help the learners to define the term 'relative speed' and discuss examples 3 and 4 as in the students' book.
- The learner should be given exercise 17.5.

Additional Hints

(i) The teacher should emphasise that the area under a velocity-time graph is equal to the distance covered.

(ii) The terms scalar and vector quantities should be avoided.

(iii) The teacher should emphasise that relative speed occurs when two bodies are moving simultaneously.

Evaluation

- Give a written test on linear motion. Discuss it with the learners after marking.
- Source related questions from elsewhere and have the learners attempt them. Discuss these thereafter, so that all concepts are grasped well.

Answers

Exercise 17.1

1. $46\frac{2}{3}$ km/h
2. $62\frac{1}{2}$ km/h
3. $8\frac{4}{7}$ km/h
4. $4\frac{1}{2}$ km/h
5. 80 km/h
6. (a) BC = 58 (b) 40 mins (c) 60 km/h

Exercise 17.2

1. 5 m/s
2. $18\frac{1}{2}$ m/s

3. O m/s
4. 22.2 s
5. 0.5 m/s
6. (a) 80 km/h (b) 111.1 m

Exercise 17.3

1. (a) (i) 80 km/h (ii) 100 km/h (iii) 130 km/h
   (iv) $112\frac{1}{2}$ km/h (v) 125 km/h (vi) 90.9 km/h
   (b) (i) 280 km (ii) 230 km
   (c) 1 p.m – 2 p.m
2. (a) 12.36 p.m (b) 5 km
3. (b) (i) $53\frac{1}{3}$ km/h (ii) 39.4 km/h
   Check for accuracy of graphs.

Exercise 17.4

1. 100 m
2. 45 m
3. (a) (i) 10 m/s (ii) 20 m/s (iii) 30 m/s (iv) 30 m/s
   (b) (i) 2 m/s^2 (ii) 4 m/s^2 (iii) O m/s^2 (vi) – 6 m/s^2
   (c) (i) 50 m (ii) 550 m (iii) 162.5 m
   (d) (i) 10 m/s (ii) 45 m/s (iii) 60 m/s
4. (a) (i) 47.5 m/s southwards (ii) 55 m/s southwards
   (iii) 15 m/s southwards
   (b) (i) 50 m/s (ii) 55 m/s
   (c) 1 500 m

Exercise 17.5

1. 9.00 a.m
2. 23 secs
3. 1.02 hours, 102.63 km
4. 7 km
5. 11.40 a.m, 280 km

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Chapter Eighteen

STATISTICS

This is not a new topic to the learner, who has been introduced to rudiments of collecting information by tabulation, bar graphs, pie charts and line graphs. He/she has also been introduced to the simple methods of calculating measures of central tendency. All these will be covered in greater detail and an extension made to include presentation of data by use of frequency polygons and histograms.

Objectives

By the end of the topic, the learner should be able to:

(i) define statistics.

(ii) collect and organise data.

(iii) draw a frequency distribution table.

(iv) calculate measures of central tendency, i.e., mean, mode, media for ungrouped and grouped data.

(v) group data into reasonable classes.

(vi) represent data in form of line graphs, bar graphs, pie-charts, pictogram, histogram and frequency polygons.

(vii) interpret data from real situations.

Time: Twenty lessons.

Teaching/Learning Activities

Definition of Statistics

- The teacher should give the learner a comprehensive definition of statistics, as in the students' book, giving appropriate examples for each term used.

Collecting and Organising Data

- The teacher should guide the learner to collect data using class statistics, as indicated in the students' book (class project).
- The teacher should lead the learner to see the need for organising the data.

Frequency Distribution Table

- The teacher should discuss the various parts of the frequency distribution table using table 18.1 in the students' book.

- The teacher should guide the learner through example 1 in the students' book.

Measures of Central Tendency for Ungrouped Data

- The teacher should involve the learner in defining the terms mean, mode and median using appropriate examples as in the students' book.
- The learner should be given exercise 18.1.

Grouped Data

- The teacher should discuss the importance of grouping data using illustration, as in the students' book.
- The teacher should help the learner to define terms range, class limits, class boundaries and class size, as in the students' book.
- Using table 18.6, the teacher should guide the learner to find the modal class, median and mean in a grouped data.
- The learner should be given exercise 18.2

Representation of Data

- The teacher should discuss various methods of representing data, as in the students' book.

Interpretation of Data from Real Life Situations

- Using the class project, the learner should be led to interpret data.

Additional Hints

(i) The teacher should lead the learner through the various methods of collecting information/data, e.g., questionnaire, interview, observation, etc.

(ii) When making a frequency distribution table, it is important that once the score/item is counted and a tally mark put, that item should be cancelled. This acts as a check to avoid repeating or missing the score.

(iii) When calculating/estimating the median in a grouped data, the formula below can be used;

$$\text{Median} = L_b + \frac{\frac{(N+1)i}{2}}{f}$$

where L_b is the lower boundary, N the total frequency, i the class interval and f the frequency of the median class.

(iv) Clearly distinguish between bar graphs and histograms. In bar graphs, bars are of uniform width while in histograms, the area of the bar is proportional to the frequency.

(v) The teacher should give further examples on emerging issues, such as gender and HIV/AIDS.

Evaluation

- Give a written test on statistics as per the Form Two scope. Discuss the paper with the learners after marking.
- Randomly challenge the learners (both oral and written) on such areas as mean, mode, median, pie-charts, pictograms etc. Ensure these are observed in full for the benefit of all learners.

Answers

Exercise 18.1

1. Mean is 9.2, mode 9, median 9.
2. (a)

No of children (x)	*Tally*	*Frequency (f)*	*fx*
1	~~////~~ ////	9	9
2	~~////~~ //// /	11	22
3	~~////~~ ////	9	27
4	////	4	16
5	~~////~~	5	25
6	///	3	18
7	//	2	14
8	///	3	24
9	/	1	9
11	/	1	11
		$\Sigma f = 48$	$\Sigma fx = 175$

(b) Mode = 2

(c) Mean = 3.65

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3.

Mass (in kg) x	Tally	Frequency f	fx
38	/	1	38
39	/	1	39
40	/	1	40
41	/	1	41
43	/	1	43
45	/	1	45
47	/	1	47
48	//	2	96
49	/	1	49
51	/	1	51
52	//	2	104
53	/	1	53
56	//	2	112
57	/	1	57
59	/	1	59
60	/	1	60
63	/	1	63
		$\Sigma f = 20$	$\Sigma fx = 997$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{997}{20} = 49.85$$

Mode 48, 52, 56.

4. (a)

Mass (in kg) x	Tally	Frequency f	fx
38	/	1	38
39	//	2	78
40	////	4	160
41	//	2	82
42	~~////~~	5	210
43	/	1	43
44	//	2	88
45	////	4	180
46	////	4	184
47	//	2	94
48	///	3	144
49	/	1	49
		$\Sigma f = 31$	$\Sigma fx = 1\ 350$

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(b) Mode is 42 kg

(c) Mean $= \frac{\Sigma xf}{\Sigma f} = \frac{1\,350}{31} = 43.54$ kg

5.

Length (cm) x	*Tally*	*Frequency* f	fx
11.8	/	1	11.8
12.5	/	1	12.5
12.8	/	1	12.8
13.4	/	1	13.4
13.5	/	1	13.5
13.6	/	1	13.6
13.7	/	1	13.7
13.9	/	1	13.9
14.3	/	1	14.3
14.4	//	2	28.8
14.5	//	2	29.0
14.7	/	1	14.7
15.2	/	1	15.2
15.3	/	1	15.3
		$\Sigma f = 16$	$\Sigma xf = 222.5$

Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{222.5}{16} = 13.91$ cm

Mode 14.4, 14.5 cm

6.

Diameter (mm) x	*Tally*	*Frequency* f	fx
10.1	/	1	10.1
10.2	/	1	10.2
19.8	/	1	19.8
19.9	///	3	59.7
20.0	////	4	80.0
20.1	~~////~~	5	100.5
20.2	/	1	20.2
20.3	~~////~~	5	101.5
20.4	~~////~~ /	6	122.4
20.5	////	4	82.0
20.6	//	2	41.2
20.7	//	2	41.4
20.8	//	2	41.6
20.9	//	2	41.8
21.0	/	1	21.0
		$\Sigma f = 40$	$\Sigma fx = 793.4$

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(b) Mean $= \frac{\Sigma xf}{\Sigma f}$

$= \frac{793.4}{40}$

$= 19.83$ mm

Mode $= 20.4$ mm

(d) Median 20.3 mm

7. Mean = 53.3, mode = 52.35, Median = 52.35

8.

Height (m) *x*	*Tally*	*Frequency* *(f)*	*fx*
1.21	/	1	1.21
1.24	/	1	1.24
1.25	/	1	1.25
1.28	//	2	2.56
1.30	/	1	1.30
1.32	/	1	1.32
1.33	/	1	1.33
1.34	/	1	1.34
1.35	/	1	1.35
1.36	///	3	4.08
1.38	///	3	4.14
1.39	//	2	2.78
1.40	///	3	4.20
1.42	////	4	5.68
1.44	///	3	4.32
1.46	/	1	1.46
1.50	////	4	6.0
1.52	/	1	1.52
1.53	//	2	3.06
1.60	//	2	3.20
1.61	/	1	1.61
1.62	//	2	3.24
1.66	/	1	1.66
1.67	/	1	1.67
1.70	//	2	3.40
1.72	//	2	3.44
1.78	/	1	1.78
1.80	//	2	3.60
		$\Sigma f = 50$	$\Sigma fx = 73.74$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{73.74}{50} = 1.47 \text{ m}$$

mode = 1.42 m, 1.50 m

$$\text{Median} = \frac{1.42 + 1.44}{2} = \frac{2.86}{2}$$

$$= 1.43 \text{ m}$$

9. Mean = 39.8, mode = 41.0, median = 41.0

10.

Volume x	*Tally*	*Frequency* f	fx
0.47	/	1	0.47
0.48	////	4	1.92
0.49	///	3	1.47
0.50	~~////~~ ///	8	4.00
0.51	//	2	1.02
0.52	///	3	1.56
0.53	//	2	1.06
0.54	/	1	0.54
		$\Sigma f = 24$	$\Sigma fx = 12.04$

$$\text{Mean} = \frac{12.04}{24} = 0.50$$

Mode = 0.50

Median = 0.50

11. Mean = 4.25, mode = 3, Median = 4
12. 46.4
13. a = 2, median is 5
14. $16\frac{22}{35}$ years or 16.6 years.
15. Mean mark for boys is 62.3.
16. Height of the 6^{th} girl was 130 cm.

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Exercise 18.2

1.

Class	*Mid-class* x	*Tally*	*Frequency* f	fx
10 – 19	14.5	~~////~~ //	7	101.5
20 – 29	24.5	~~////~~	5	122.5
30 – 39	34.5	~~////~~ ~~////~~ ///	13	448.5
40 – 49	44.5	~~////~~ ////	9	400.5
50 – 59	54.5	~~////~~ ~~////~~ ///	13	708.5
60 – 69	64.5	~~////~~ ~~////~~ ////	14	903.0
70 – 79	74.5	~~////~~ /	6	447.0
80 – 89	84.5	///	3	253.5
90 – 99	94.5	//	2	189.0
			$\Sigma f = 72$	$\Sigma fx = 3\,574$

(a) modal class 60 – 69

(b) (i) Estimated mean $= \frac{3\,574}{72}$

$= 49.6$

(ii) Median 36th student $= 49.5 + \frac{2}{13} \times 10 = 49.5 + \frac{20}{13}$

$= 51.038$

37th student $49.5 + \frac{3}{13} \times 10 = 49.5 + \frac{30}{13}$

$= 51.808$

Median $= \frac{51.038 + 51.808}{2}$

$= 51.423$

$= 51.4$

2. (a)

Class	*Mid-class* x	*Tally*	*Frequency* f	fx
135 – 144	139.5	~~////~~ /	6	837
145 – 154	149.5	~~////~~ /	6	897
155 – 164	159.5	~~////~~ ////	9	1 435.5
165 – 174	169.5	~~////~~ ~~////~~ /	11	1 864.5
175 – 184	179.5	~~////~~ ~~////~~ ////	14	2 513
185 – 194	189.5	////	4	758
			$\Sigma f = 50$	$\Sigma fx = 8\,305$

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(b) Modal class 175 – 184

Median 168.6 cm (ii) Mean 166.1 cm

25^{th} student $164.5 + \frac{4}{11} \times 10 = 164.5 + 3.636$

$= 168.136$

26^{th} student $164.5 + \frac{5}{11} \times 10 = 169.045$

Median $= 168.6$

3. Range $= 190 - 86$

$= 104$

Class size 10

No. of classes $= \frac{104}{10}$

$= 10.4$

$\simeq 11$

Class	*Mid class (x)*	*Tally*	*Frequency (f)*	*fx*
85 – 94	89.5	~~////~~	5	447.5
95 – 104	99.5	////	4	398.0
105 – 114	109.5	/	1	109.5
115 – 124	119.5	////	4	478.0
125 – 134	129.5	////	4	518.0
135 – 144	139.5	////	4	558.0
145 – 154	149.5	/	1	149.5
155 – 164	159.5	~~////~~ //	7	1116.5
165 – 174	169.5	////	4	847.5
175 – 184	179.5	~~////~~	4	718.0
185 – 194	189.5	///	3	568.5
			$\Sigma f = 42$	$\Sigma fx = 5909$

(a) Modal class 155 – 164

(b) (i) Estimated mean $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{5909}{42}$

$= 140.7$

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(ii) Estimated median, 21^{st} district $= 134.5 + \frac{3}{4} \times 10$
$= 134.5 + 7.5$
$= 142$

22^{nd} district $= 134.5 + \frac{4}{4} \times 10$
$= 144.5$

$$\text{Median} = \frac{142 + 144.5}{2}$$
$$= 143.3$$

4.

Class	*mid class (x)*	*Tally*	*Frequency (f)*	*fx*
25 – 29	27	/	1	27
30 – 34	32	//	2	64
35 – 39	37	///	3	111
40 – 44	42	///	3	126
45 – 49	47	~~////~~	5	235
50 – 54	52	////	4	208
55 – 59	57	~~////~~ //	7	399
60 – 64	62	~~////~~ /	6	372
65 – 69	67	//	2	134
70 – 74	72	////	4	288
75 – 79	77	/	1	77
80 – 84	82	//	2	164
			$\Sigma f = 40$	$\Sigma fx = 2\ 205$

(i) Mean $= \frac{\Sigma fx}{\Sigma f}$
$= \frac{2\ 205}{40}$
$= 55.1$

(ii) Median; 20^{th} student $= 54.5 + \frac{2}{7} \times 5 = 55.929$
21^{st} student $= 54.5 + \frac{3}{7} \times 5 = 56.643$

$$\text{Median} = \frac{55.929 + 56.643}{2}$$
$$= 56.3$$

(iii) Modal class 55 – 59

5. (i) Mean = 18.01 kg
(ii) Median = 17.2 kg
(iii) Modal class (10 – 14) kg

6. (i) Mean = 38.6
(ii) Median = 44.0
(iii) Modal class 50 – 59

7. (i) Mean = 33.6 mm
(ii) Median 33.4 mm
(iii) Modal class = 33 – 34

8. (i) Mean = Sh. 588.70
(ii) Median = Sh. 392.90
(iii) Modal class = Sh. (282 – 372)

9.

Class	*Mid class* x	*Tally*	*Frequency* f	fx
10 – 14	12	𝍸 ///	8	96
15 – 19	17		0	0
20 – 24	22	𝍸 /	6	132
25 – 29	27	////	4	108
30 – 34	32	//	2	64
35 – 39	37	///	3	111
40 – 44	42	𝍸	5	210
45 – 49	47	////	4	188
50 – 54	52	////	4	208
55 – 59	57	////	4	228
			$\Sigma f = 40$	$\Sigma fx = 1\,345$

(a) Modal class is 10 – 14

(b) (i) Mean $= \frac{1345}{40}$

$= 33.6$

(ii) Median = 20[th] item $29.5 + \frac{0}{2} \times 5 = 29.5$

21[st] item $34.5 + \frac{1}{3} \times 5 = 36.17$

Median $= \frac{29.5 + 36.17}{2}$

$= 32.8$

10.

Class	*Mid score* x	*Tally*	*Frequency* f	fx
11.0–11.4	11.2		0	0
11.5–11.9	11.7	~~////~~ /	6	70.2
12.0–12.4	12.2	~~////~~ /	6	73.2
12.5–12.9	12.7	////	2	25.4
13.0–13.4	13.2	~~////~~	5	66.0
13.5–13.9	13.7	////	2	27.4
14.0–14.4	14.2	/	1	14.2
14.5–14.9	14.7	~~////~~ ~~////~~ /	11	161.7
15.0–15.4	15.2	~~////~~ ~~////~~ ////	14	212.8
15.5–15.9	15.7	///	3	47.1
			$\Sigma f = 50$	$\Sigma fx = 698.0$

(a) Modal class 15.0 – 15.4

(b) (i) Mean $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{698}{50}$

$= 13.96$

(ii) Median, determine 25th athlete $= 14.45 + \frac{3}{11} \times 0.4$

$= 14.56$

26th athlete $= 14.45 + \frac{4}{11} \times 0.4$

$= 14.60$

Median $= \dfrac{14.56 + 14.60}{2}$

$= 14.58$

Exercise 18.3

1. Coffee angle $= \frac{4}{24} \times 360^0$, $= 60^0$

Grass angle $= \frac{3}{24} \times 360^0$ $= 45^0$

Maize angle $= \frac{7}{24} \times 360^0$, $= 105^0$

Bananas angle $= \frac{5}{24} \times 360^0$ $= 75^0$

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Homestead angle $= \frac{0.5}{24} \times 360^0$

$= 7.5^0$

Vegetable and Pats angle $= \frac{4.5}{24} \times 360^0$

$= 67.5^0$

Check for accuracy of pie chart.

2. Check for accuracy of pie chart.
   Grade A, 45^0
   Grade B, 108^0
   Grade C, 135^0
   Grade D, 45^0
   Grade F, 27^0
   (b) (i) Percentage rep. Grade A $= \frac{5}{40} \times 100\%$
   $= 12.5\%$
   (ii) Percentage rep. Grade C and 2 $= \frac{20}{40} \times 100\%$
   $= 50\%$

3. Check for accuracy of graph.
4. Check for correct use of pictograms.
5. Check for accuracy of graph.

6.

Class	*Mid-class* x	*Boys Frequency* f	*Girls Frequency* f
115 – 119	117	1	1
120 – 124	122	2	2
125 – 129	127	2	4
130 – 134	132	4	3
135 – 139	137	4	4
140 – 144	142	2	2
145 – 149	147	1	1
150 – 154	152	3	2
155 – 159	157	0	1
160 – 164	162	1	0

6. Check for accuracy of graphs.
7. Check for accuracy of graph.
8. Range $= 44 - 24 = 20$

   Class size of 3, No. of classes $\frac{20}{3} \simeq 7$

Class	*Mid class* x	*Tally*	*Frequency* f
24–26	25	/	1
27–29	28	///	3
30–32	31	////	4
33–35	34	////	4
36–38	37	////	4
39–41	40	//	2
42–44	43	//	2

   Check for accuracy of graphs.

9. Walking: Angle $= \frac{17}{42} \times 360^0$
   $= 145.7^0$

   Cycling: Angle $= \frac{5}{42} \times 360^0$
   $= 42.9^0$

   Matatu: Angle $= \frac{18}{42} \times 360^0$
   $= 154.3^0$

   Bus: Angle $= \frac{2}{42} \times 360^0$
   $= 17.1^0$

   Check for accuracy of pie chart.

10,11. Check for accuracy of graphs.

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Chapter Nineteen

ANGLE PROPERTIES OF A CIRCLE

The learner has been introduced to the use of geometrical instruments. The teacher should revise the terms related to the circle such as arc, chord, segment and sector.

Objectives

By the end of the topic the learner should be able to:

(i) identify an arc, chord and a segment.

(ii) relate and compute angles subtended by an arc at the centre and on the circumference.

(iii) state the angle in a semicircle.

(iv) find and compute angles of a cyclic quadrilateral.

Time: Nineteen lessons.

Teaching/Learning Activities

Arc, Chord and Segment

- The teacher should define the terms arc, chord and segment using illustrations as in the students' book.

Angles Subtended at the centre and on the Circumference by the Same Arc

- Using illustrations, the learner should be led to identify the relationship between angle subtended at the centre and angle subtended on the circumference by the same arc as in the students' book.
- The teacher should lead the learners through examples 1 and 2.

Angle Subtended by the Diameter at the Circumference

- This could be established in two ways;
  (i) By drawing and measuring.
  (ii) By treating it as a case of the relationship of angle at the centre and angle on the circumference.
- The learner should be given exercise 19.1 in the students' book.

Angles in the Same Segment

- The learner should be guided to find the relationship between angles subtended by the same arc in the same segment on the circumference, as in figure 19.18.

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- The teacher should guide the learner through example 3.
- The learner should be given exercise 19.2.

Cyclic Quadrilaterals

- The learner should be guided to establish the properties of a cyclic quadrilateral as in the students' book. This should be done using the angle properties of a circle covered earlier.
- The learner should be led through example 4 and given exercise 19.3.

Additional Hints

(i) Using illustrations, the teacher should discuss the relationship of angles subtended by equal arcs on the circumference.

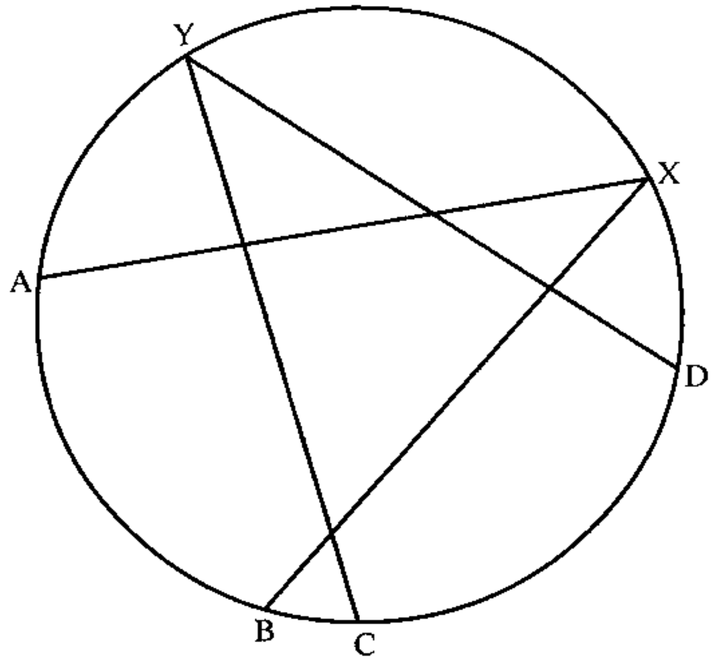

Fig 19.1

Thus, if arc AB = arc CD, $\angle AXB = \angle CYD$

(ii) When solving questions on angle properties of a circle, for each angle found, the reason must be given.

Evaluation

- Give a written test on angle properties of a circle as per the Form Two scope. Discuss the same with the learners after marking.
- Randomly present questions (oral, written) on such areas as angles of a cyclic quadrilateral and angle in a semicircle. Have the learners attempt them and thereafter a discussion of the same.

Answers

Exercise 19.1

1.

Angle at the centre			85		45.2	124		220		
Circle on the circumference	40			61					90	135

2. 100°
3. $a = 70^\circ$ $b = 220^\circ$ $c = 140^\circ$ $d = 110^\circ$
6. 111°
7. $x = 60^\circ, y = 60^\circ$
8 (a) 80° (b) 40°
12 (a) 102° (b) 108° (c) 78°
13. (a) 45° (b) 15°
14. $P = 60^\circ, t = 30^\circ$

Exercise 19.2

1. $a = 43^\circ, b = 43^\circ$
2. $V = 60^\circ, W = 30^\circ, X = 50^\circ, y = 40^\circ, Z = 50^\circ$
3. $P = 40^\circ, q = 10^\circ, r = 40^\circ, S = 40^\circ$
4. $X = 30^\circ$ $y = 40^\circ$
5. 73°
6. (a) $a = 43^\circ$ $b = 40^\circ$ $c = 25^\circ$ e 72° $f = 72^\circ$
   (b) $v = 53^\circ$ $w = 48^\circ$ $x = 32^\circ$ $y = 53^\circ$ $z = 47^\circ$
   (c) $p = 17^\circ$ $q = 96^\circ$ $r = 17^\circ$
7. 125°
8. $x = 120^\circ, y = 60^\circ$
9. **Note:** The resulting cyclic quadrilateral should be named WXZY in that order.

Exercise 19.3

1. (a) $x = 30$ $y = 60$
   (b) $x = 40^\circ$ $y = 60^\circ$
   (c) $x = 60^\circ$ (d) $x = 70^\circ, y = 60^\circ$ (e) $x = 140^\circ = 100^\circ$
2. 28^0 3. (a) 78^0 (b) 78^0
4. 40 5. 143
6. $\angle CD = 620$ $\angle ADt = 121^0$
7. $\angle ZXY = 40^0$

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Chapter Twenty

VECTORS (1)

The term 'vector' might not be a new concept to the learner, as he/she have met it in Form One Physics.

The teacher should probe the learner's understanding of the concept 'vectors'. The teacher should also revise lengths, angles and co-ordinate systems, as they will be used in this topic.

Objectives

By the end of the topic the learner should be able to:

(i) define vectors and scalars.
(ii) use vector notation.
(iii) represent vectors both singly and combined geometrically.
(iv) identify equivalent vectors.
(v) add vectors.
(vi) multiply vectors by scalars.
(vii) define column and position vectors.
(viii) find magnitude of a vector.
(ix) find the midpoint of a vector.
(x) define translation as a transformation.

Time: Twenty lessons.

Teaching/Learning Activities

Vectors and Scalars

- The teacher is advised to use familiar examples to define vector and scalar quantities. The learner should be guided in listing examples of vector and scalar quantities.
- The teacher should use the illustrations in the students' book to discuss vectors and scalars.

Vector Notation

- The teacher should discuss vector notation as in the students' book
- The teacher is advised to explain use of the symbol to denote the magnitude (modules) of a vector as shown in the students' book.

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Vector Representation

- The teacher should guide the learner through the use of directed lines to represent vectors.
- The teacher should also highlight the use of negative signs in representing vectors.

Equivalent Vectors

- The teacher should introduce and discuss equivalent vectors as illustrated in the students' book.
- The learner should be guided in finding equivalent vectors as in figure 20.5.

Addition of Vectors

- The teacher should discuss vector addition, as illustrated in the students' book.
- The learner should be guided in finding the resultant displacement, as in figure 20.7.
- The learner should be given exercise 20.1.

Multiplication of a Vector by a Scalar

- The teacher should discuss multiplication of a vector by a scalar, as in the students' book.
- The learner should be guided through example 1.
- The learner should be given exercise 20.2.

Column and Position Vector

- The teacher should introduce column vectors as shown in figure 20.20.
- The learner should be led through examples 2 and 3.
- The teacher should lead the learner through position vector, as illustrated in the students' book.
- The learner should then be given exercise 20.3.

Magnitude and Midpoint of a Vector

The learner should be led in finding the length of **AB** in figure 20.27.

- The learner should be involved in finding the magnitude of vector (i), (ii), (iii) and (iv) at the end of sub-topic 20.8, using the formula given.

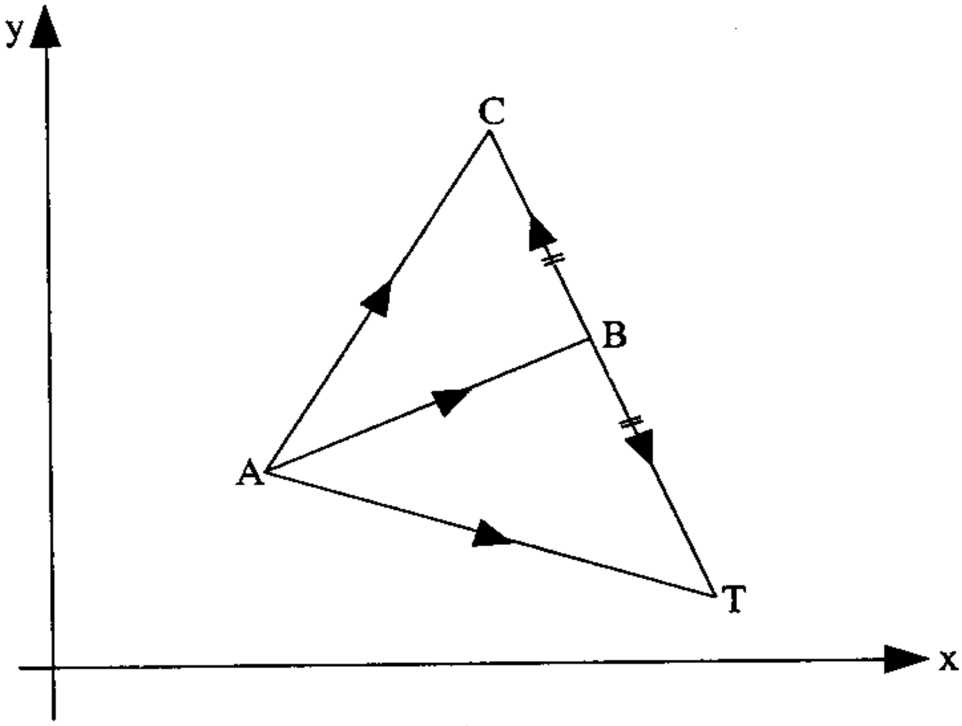
- The teacher should discuss the midpoint of a vector as in figure 20.28 and 20.29.
- The learner should be given exercise 20.4.

Translation

- The teacher should assist the learner to define translation.
- The learner should be guided through example 4 in finding translation vector, co-ordinates of the objects and co-ordinates of the image.
- The learner should be given exercise 20.5.

Additional Hints

(i) The teacher should discuss subtraction of vectors as an addition of negative vectors, as shown in the figure below:



AT = AB + BT; but BT = CB

AT = AB + (–BC)

= AB – BC

(ii) The teacher should discuss the sum of a vector and its negative to define a null vector.

(iii) Position vectors can also be defined as special column vectors with one point as the origin:

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Thus, OP $= \binom{x}{y} - \binom{0}{0}$

$= \binom{x}{y}$

Where $\binom{x}{y}$ are the x and y co-ordinates of P and $\binom{0}{0}$ are the x and y co-ordinates at the origin.

Evaluation

- Give a written test on vectors as per the Form Two scope. Have the paper discussed together with the learners after marking.
- Give the learners related questions from other sources. Have these discussed after the learners attempt them.
- Give the learners mixed exercise 3 and revision exercises 1 to 8, to provide a good summary of Form Two work. Ensure all these questions are discussed in detail after marking.

Further Questions

1. OPQ is a triangle in which **OP** = **p** and **OQ** = **q**. X and Y lie on **PQ** and **OQ** respectively such that **PX** : **XQ** = 1 : 3 and **OY** : **YQ** = 2 : 3. Find **OX** and **PY** in terms of **q** and **p**.

Answers

Exercise 20.1

1. **c** and **f**; **m** and **q**
2. **b** and **d**; **c** and **j**; **e** and **n**; **h** and **I**; **h** and **p**
3. Check for accuracy of drawings.
4. **AC** = **b** + **c**   **AH** = **a** + **b** + **c**
   AG = **b** + **a**   **BE** = **c** – **b** + **a**
   BD = **c** – **b**

Exercise 20.2

1. (a) **a** (b) **a** – 2**b** (c) 2**a** – 2**b**
   (d) **a** – **b** (e) **b** – **a** (f) **a** + **b**
2. (a) (i) $\mathbf{q} = \frac{3}{2}\mathbf{q}$ (ii) $\mathbf{p} = \frac{2}{3}\mathbf{q}$
   (b) (i) $\mathbf{q} = 2\mathbf{p}$ (ii) $\mathbf{p} = \frac{1}{2}\mathbf{q}$
   (c) (i) $\mathbf{q} = \frac{5}{3}\mathbf{p}$ (ii) $\mathbf{p} = \frac{3}{5}\mathbf{p}$

(d) (i) $\mathbf{q} = O\mathbf{p}$ (ii) No relationship

3. (a) $4\mathbf{a} - 2\mathbf{b}$ (b) $\frac{1}{2}\mathbf{b} + \frac{3}{2}\mathbf{a}$ (c) $\mathbf{b} - 2\mathbf{a}$

   (d) $-2\mathbf{b} - 6\mathbf{a}$ (e) $-2\mathbf{a} - 4\mathbf{b}$ (f) $\frac{3}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

   (g) $3\mathbf{b} + 4\mathbf{a}$
4. Check for accuracy of drawings.
5. Check for accuracy of drawings.
6. (a) $-\mathbf{a}$ (b) $\mathbf{a} + \mathbf{b} + \mathbf{c}$ (c) $\mathbf{a} - \mathbf{b} - \mathbf{c}$

   (d) $-\mathbf{b} - \mathbf{c}$ (e) $\mathbf{c} - \mathbf{a}$ (f) $\mathbf{a} - \mathbf{c}$

   (g) $\mathbf{b} + \mathbf{a}$

7. (a) $-17\mathbf{a} - 2\mathbf{b}$ (b) $-\frac{2}{3}\mathbf{a} + \frac{5}{12}\mathbf{b}$

8. $\mathbf{a} = -\frac{3}{5}\mathbf{b}$

9. $\mathbf{y} = -\frac{5}{2}\mathbf{x}$ $\mathbf{z} = \frac{3}{2}\mathbf{x}$
10. (a) $7\mathbf{a} - \mathbf{b} - 7\mathbf{c}$ (b) $3\mathbf{u} + 2\mathbf{w}$ (c) 0

11. (i) $\frac{3}{2}\mathbf{a} + \mathbf{b}$ (ii) $-\frac{3}{2}\mathbf{a} + 3\mathbf{c}$ (iii) $\mathbf{b} + 3\mathbf{c}$

    (iv) $3\mathbf{c} + \mathbf{b}$ (v) $\frac{3}{2}\mathbf{a} + \mathbf{b} + \mathbf{c}$ (vi) $\frac{3}{2}\mathbf{a} + \frac{1}{3}\mathbf{b} + 2\mathbf{c}$

    (vii) $\frac{1}{2}\mathbf{a} + \frac{2}{3}\mathbf{b} + 3\mathbf{c}$ (viii) $-\frac{1}{2}\mathbf{a} + \mathbf{c}$ (ix) $\frac{1}{2}\mathbf{a}$

    (x) $\mathbf{a} + \mathbf{b} - \mathbf{c}$ (xi) $-\frac{1}{2}\mathbf{a} + \frac{2}{3}\mathbf{b} + 2\mathbf{c}$ (xii) $-2\mathbf{a} - \frac{1}{3}\mathbf{b}$

    (xiii) 0 (xiv) $2\mathbf{c} - \frac{3}{2}\mathbf{a} - \mathbf{b}$ (xv) $2\mathbf{a} + \frac{2}{3}\mathbf{b}$

    (xvi) $\frac{1}{2}\mathbf{a} + \frac{1}{3}\mathbf{b} + \mathbf{c}$

Exercise 20.3

1. $\mathbf{AB} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\mathbf{TS} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\mathbf{ED} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$ $\mathbf{IH} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\mathbf{GF} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

   $\mathbf{JK} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\mathbf{QR} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $\mathbf{NP} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$ $\mathbf{CO} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $\mathbf{LM} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

   $\mathbf{OF} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\mathbf{OC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\mathbf{OQ} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$
2. Check for accuracy of graph.

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3. $\mathbf{OE} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\mathbf{OC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\mathbf{OQ} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ $\mathbf{OL} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ $\mathbf{OP} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

$\mathbf{OI} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

4. (a) (i) $\begin{pmatrix} 13 \\ -11 \end{pmatrix}$ (ii) $\begin{pmatrix} 19 \\ 32 \end{pmatrix}$ (iii) $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$

(iv) $\begin{pmatrix} 0 \\ 26 \end{pmatrix}$ (v) $\begin{pmatrix} \frac{7}{3} \\ \frac{13}{3} \end{pmatrix}$ (vi) $\begin{pmatrix} -10 \\ 0 \end{pmatrix}$

(vii) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$r = -32, s = 19$

5. $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ 6. (3, 12)

7. $\lambda = \dfrac{-3}{5}$ $\mu = \dfrac{-6}{5}$

Exercise 20.4

1. (a) (–1, 1.5) (b) (0, 1)
   (c) (0.5, 2) (d) $(\frac{1}{2}a, \frac{1}{2}b)$
2. (a) (i) (1, 4) D(5, 1)
   (b) (i) 5 units (ii) 10 units
3. (a) $\frac{2}{3}\mathbf{b}$ (b) $\frac{1}{3}\mathbf{b} + \frac{2}{3}\mathbf{a}$ (c) $OE = \frac{4}{3}\mathbf{a}$
   (d) $\frac{2}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$ (e) $\frac{2}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$

Exercise 20.5

1. $B' = (-2, 4)$ $C' = (0, -1)$ $D' = (-1, -3)$
2. Plot $P'(-1, 3)$, $Q'(1, 1)$ $R'(0, 0)$

3. $P' = (-3, 7)$ $P'' = (-2, 5)$ $T = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

4. $T_1 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $A''(4, 6)$ $T_3 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

5. (a) (6, 6) (b) (–2, 1) (c) (2, –1)
   (d) (0, 0) (e) (–1, –1) (f) (–2, 2)

6. (a) (1, 2) (b) (–4, 3) (e) (7, 14)
   (d) (7, 10) (e) (12, –4) (f) (–9, –1)
   (g) (7, 13) (h) (7, –11) (i) (4, 3)
   (j) (a – c, b – d)

7. Plot A'(8, 5) B'(7, 8) C'(5,5)

8. $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

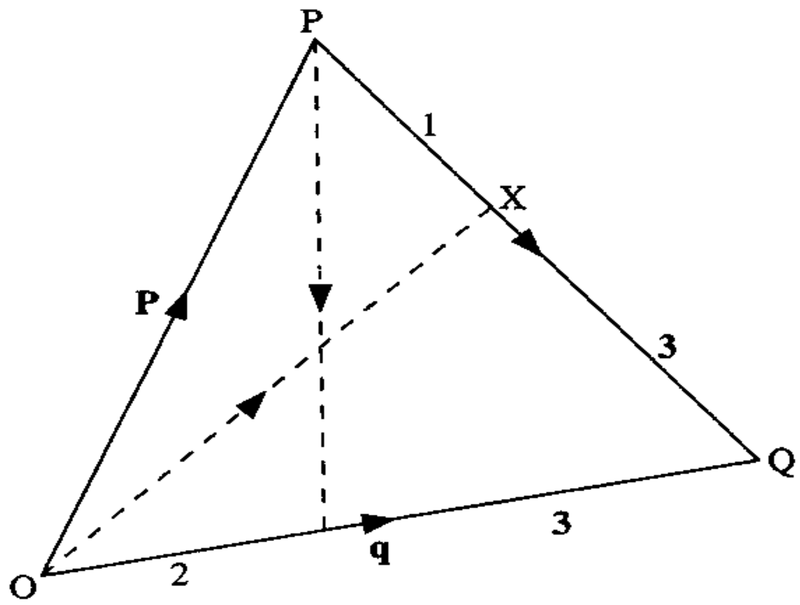
9. $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

10. (a) Plot the points

(i)	P′(2, 3)	Q′(5, 5)	R′(2, 7)	S′(3, 5)
(ii)	P′(0, –6)	Q′(3, –4)	R′ (0, –2)	S′(1, –4)
(iii)	P′(5, –1)	Q′(8, 1)	R′(5, 3)	S′(6, 1)
(iv)	P′(–3, –5)	Q′(0, –3)	R′(–3, –1)	S′(–2, –3)

(b) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$



Further Questions

PX : **XQ** = 1 : 3

Total ratio = (1 + 3)

= 4

$\therefore \mathbf{PX} = \frac{1}{4}\mathbf{PQ}$

$= \frac{1}{4}(-\mathbf{p} + \mathbf{q})$

$= \frac{1}{4}(\mathbf{q} - \mathbf{p})$

Therefore **OX** = **OP** + **PX**

$= \mathbf{p} + \frac{1}{4}(\mathbf{q} - \mathbf{p})$

$= \mathbf{p} + \frac{1}{4}\mathbf{q} - \frac{1}{4}\mathbf{p}$

$= \frac{3}{4}\mathbf{p} + \frac{1}{4}\mathbf{q}$

$= \frac{1}{4}(3\mathbf{p} + \mathbf{q})$

OY : **YQ** = 2 : 3

Total ratio = (2 + 3) = 5

$\therefore \mathbf{OY} = \frac{2}{5}\mathbf{OQ} = \frac{2}{5}\mathbf{q}$

Therefore, $\mathbf{PY} = \mathbf{PO} + \mathbf{OY} = -\mathbf{p} + \frac{2}{5}\mathbf{q}$

$\mathbf{PY} = \frac{2}{5}\mathbf{q} - \mathbf{p}$

Mixed Exercise 3

1. (a) Check for accuracy of graph.
   (b) (i) 5 m/s^2
   (ii) $\frac{5}{3} \text{ m/s}^2$
   (iii) 80 m
2. (a) $m = 2$
   (b) mode = 17
3. Mean = 641.7, Mode = 633, Median = 633
4. 495 g
5. 80
6. (a) 64.7 (b) Check for correct diagram.
7. (a) 2.5 (b) 0.4
   (c) Check for accuracy.
8. 60 hectares
9. (b) Mean = 16.5; Median = 168.42
10. $n = 4$
11. 34
12. (a) Modal class is 105 – 109
    (b) Mean = 128.24
13. $(-5, -10)$
14. (i) $\mathbf{OA} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$, $\mathbf{OC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
    (ii) $\mathbf{AB} = \sqrt{13}$ $\mathbf{AC} = \sqrt{52} = 2\sqrt{13}$
15. $P'(-9, 4)$ $Q'(-6, 7)$ and $R'(-4, 4)$
16. $C = -e$
17. $x = 4$ or -2
18. (a) $\frac{7a + 3b}{2 + 6}$ (b) $4x + 2m$ (c) $r + k$

20. $RP = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$ $PR = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

    (a) $P'Q = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $Q'R' = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $P'R' = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$

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(b) $P'Q' = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $Q'R' = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$ $R'P' = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

(c) $P'Q' = \begin{pmatrix} -0.8 \\ 4.4 \end{pmatrix}$ $Q'R' = \begin{pmatrix} -4.2 \\ 0.6 \end{pmatrix}$ $P'R' = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$

21. $x = 6$ or -3

22. $P''(-3, 8)$ $T_3 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

23. (a) $\begin{pmatrix} -34 \\ -9 \end{pmatrix}$ (b) $\begin{pmatrix} -12 \\ -2.5 \end{pmatrix}$ (c) $x = -3$

$y = -2$

24. (a) $-e$ (b) $a + b$ (c) $a + b + e$ (d) $b - e$

(e) $a + b - \frac{1}{2}e$ (f) $\frac{1}{2}a + \frac{1}{2}b - e$ (g) $\frac{1}{2}(a + b + e)$

25. (a) $(3x + y)\ (x + 3y)$ (b) $(x - 1)\ (x + 3)\ (x^2 - 2x + 3)$

(c) $3x\ (x + 2y)$ (d) $(x - 3)\ (x + 1)\ (x^2 + 2x + 3)$

26. $P'(3.2, 1.6)$, $Q'(6, 2)$ $R'(7, 5)$ and $P''(0.2, 3.6)$

$Q''(3, 4)$ $R''(4, 7)$

27. $a = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

28. 43.67 cm

29. Squares, 4 ha.

30. 30^0

31. (a) $\angle DAC = 35^\circ$ (b) $\angle BCA = 30^\circ$ (c) $\angle BDC = 75^\circ$

32. (a) 40° (b) 60°

33. 50°

34. $\angle NRM = 50^\circ$, $\angle QNP = 20^\circ$

35. $\angle RSO + \angle SRQ = 180^0$ $\angle SPQ + \angle SRQ = 180^\circ$

36. $\angle ADC = \angle CBO$

37. 200°

38. (a) (i) $0 < x < 1$ (ii) $x > 1$ or $x < 0$

(b) (i) $x < 0$ (ii) $x > 0$

(c) (i) $0 < x < 1$ (ii) $x > 1$ or $x < 0$

39. $y < x$, $y \leq -x$, $2y + x + 6 \leq 0$

40. 29°
41. $-4.5 \leq y < 20$
42. 18.2 square units
43. 4 or -1
44. $2y \leq x$, $y \leq -x$, $y \geq -7$, 49 square units
45. $y \geq 2x - 8$
46. $y \geq 0$, $x \geq 0$, $y = 4$, $x \leq 3$
47. (0, 5), (1, 2), (1,3), (1, 4), (2, 3), (2, 4), (3, 4); 4 square units
9.15 units
48. 7.5 square units
49. Check for accuracy of graph.
50. (a) 50
(b) 275 m
(c) 1 840 m
(d) 8th, 19 m/s^2
12th, 18 m/s^2

(e) $\frac{1}{3}$ m/s^2

Revision Exercise 1

1. 1.823
2. Equation of line of symmetry, $y = x$
(a) A'(0, 0) B'(–5, 2) C' (–9, 9) D'(–2, 5) (b) $y = -x$
3. (a) $\angle RPQ = \angle PRS$, and $\angle PSR = \angle PRQ$
(b) PR = 12 cm
4. (a) $\begin{pmatrix} 6 \\ 8\frac{1}{2} \end{pmatrix}$ (b) $\begin{pmatrix} -14 \\ -16 \end{pmatrix}$ (c) $\begin{pmatrix} 6\frac{1}{8} \\ 8\frac{31}{32} \end{pmatrix}$
5. (a) 602 cm^2 (b) 869.3 cm^3
6. Show correct representation.
7. (a) (i) Grad Ab = grad CD = –3
(ii) Grad AD = grad CB = $-\frac{1}{3}$
(iii) Grad AC x grad DB = –1 x 1 = –1
(b) E (3, 3)
8. (a) 7*l* (b) 5*l* (c) Check correct histogram
9. $x^2 + y^2 + z^2$
10. (a) (7, 4) (b) (7, 3) (c) (10, 2)

Revision Exercise 2

1. (a) (9, 3) (b) (2, 6) (c) (10, 3) (d) (10, –2)
2. (a) $y = -\frac{1}{2}x$ (b) $y = 0$
   (c) $y + x + 1 = 0$ (d) $y = -2x + 8$
3. 2, 10, 8
4. 236
5. (a) $2(2x - 3y)(x + z)$ (b) $(3p - 2)(p - 2)$
   (c) $(s + 2t)(2s - t)$ (d) $(8k - 7n)(8k + 7n)$
6. (a) 48 cm^2 (b) $\sqrt[4]{3}$ cm
7. Check for correct figures
8. (a) 4.5 cm (b) 890 cm^3 (nearest whole number)
9. $r_1 = 2$ cm, $r_2 = 5$ cm, 131.3π cm^2

Revision Exercise 3

1. (a) Check for correct frequency distribution table
   (b) x = 11.56
   (c) Median 11.5
2. P′(–3, –1), Q′(–8, –1), R′(–5, –3)
3. (4, 4), (2, 2), (5, 1)
4. $y \geq \frac{1}{4}x + 1$, $y < \frac{4}{3}x + 4$, $y \leq -4x + 4$
5. 784
6. (a) 1 408 cm^2 (b) 3 080 cm^3
7. A″(2, –6), B″(3, –4), C″(2, –2) D″(0, –2), E″(–1, –4), F″(0, –6)
8. (a) A′(7, 4), B′(7, 9), C′(5, 8), D′(3, 9), E′(3, 4), F′(5,5)
   (b) A′(4, –2), B′(4, –4.5), C′(5,–4), D′(6, –4.5), E′ (6, –2), F′(5, –2.5)
9. P = 0, or p = –5

Revision Exercise 4

1. (a) $-\frac{3}{2}$ (b) –2 (c) $-\frac{2}{3}$ (d) Not defined (e) 0
2. 1 : 3
3. Scale factor –1, centre (5, –7)
4. N(3, 4); M(5.5, 5.5), D(4.5, 4.5)

5. $\lambda = \frac{2}{5}$, $\mu = \frac{1}{5}$
6. $S = 2\,464$ cm^2, $V = 11\,500$ cm^3
7. (a) P_1 and P_3, P_2 and P_4 (b) P_1, and P_2, P_2 and P_3
   P_3 and P_4 ; P_4 and P_1
8. $34 \leq x \leq 50$
9. (a) 3.07 (b) 3.538
10. 6.5 cm, 42.25 cm^2

Revision Exercise 5

1. Check on correct diagram
2. Several approaches
3. $2149\frac{5}{7}$ cm^3
4. $|PQ| = |QR| = \sqrt{17}$
5. 644
6. (a) 27:64 (b) 52.7 cm^3
7. Grad AC = 0 $\Rightarrow$ line AC is horizontal
   Grad BC = ∞ $\Rightarrow$ line BC is vertical
   Line AC is perpendicular to BC
   Area = 13.5 cm^2
8. $x = 3$, $y = 5$
9. (a) 71.6° (b) 45° (c) 33.7°
10. $6x(3x - 4y)$

Revision Exercise 6

1. (a) Check for accurate drawing of histograms
   (b) Check for accurate drawing of frequency polygons
   (c) English 40.25, Maths 46.25
   (d) Performance in Maths is better than in English
2. 1.3 *l*
3. $y = x - 1$
4. (1, 5); (2, 4); (2, 5); (2, 6); (3, 3), (3,4), (3, 5); (3, 6) and (3.7)
5. Check for correct facts
6. 26.8 cm^2
7. 32 cm^2
8. On the second day at 12.35 a.m.

9. $a = \pm\sqrt{17}$
10. $P'(2, -1)$, $Q'(2, -3)$, $R'(4, -3)$; $S'(4, -1)$

Revision Exercise 7

1. 3.95 cm
2. $P'(-4, 7)$, $Q'(-4, 1)$; $R'(-13, 4)$; $P''(2, -3)$, $Q''(2, 3)$; $R''(11, 0)$
3. (a) $(-1, 6)$ (b) area = 9 square units
4. Check for correct proof
5. $81.1\ cm^2$
6. Check for correct proof.
7. Δs EAB and ECD, etc. $y = 12, u = 12$
8. $A'''(-6, 4)$, $B'''(-4, 5)$, $C'''(-4, 3)$
9. $P''(-3, -3)$. $Q''(-6, -3)$, $R''(-4.5, -6)$
10. $P = -4$, or $P = 1$, $x + 2y = -4$ or $x + 2y = 1$

$X = 2\frac{1}{2}$ and $y = -\frac{3}{4}$, or $x = 0$, and $y = -2$

Revision Exercise 8

1. 1.079
2. 16
3. $140\ cm^2$
4. 6 square units, $y \le 0$, $y \ge x - 2$ and $y + 3x > -6$
5. 1.327
6. $32.1\ cm^3$
7. $-\frac{1}{ac}$, -51
8. (a) 2 (b) (i) $y = 2x + 7$ (ii) $2y = -x + 8$
9. 216 000 litres
10. (a) Profit sh. 6 on towel, loss of sh. 5 on blanket
    (b) 512 towels.